Recent Advances in Experiments and Modeling of Grid-forming Systems

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May 25th, 2023

The University of Texas at Austin
Chandra Department of Electrical and Computer Engineering
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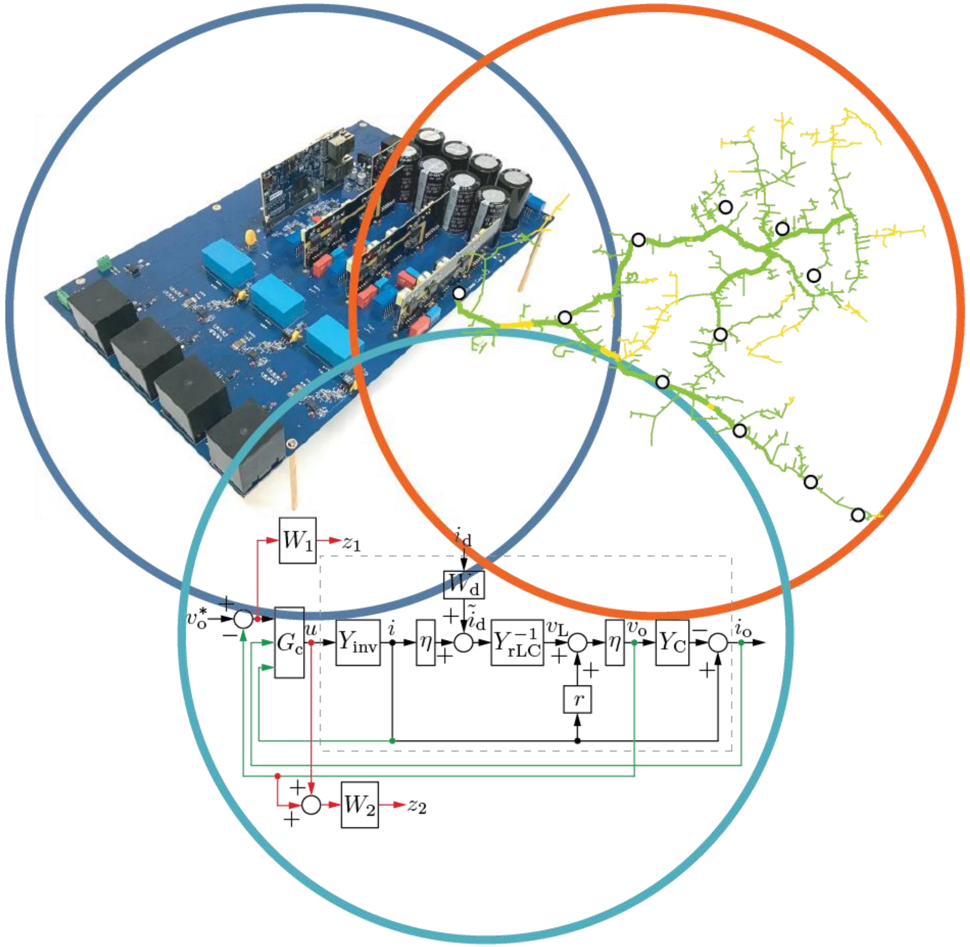
My Lab Activities

Power Electronics

Power Systems



Control Systems

Acknowledgements

Contributions from my postdoc and 6 grad students


Minghui


Kamakshi


Rahul


Weiqian


Soham


Pranav


Debjyoti

and generous support from DOE & NSF for my research on:

- Grid-forming systems
- Power electronics
- UNIFI Consortium

SOLAR ENERGY TECHNOLOGIES OFFICE
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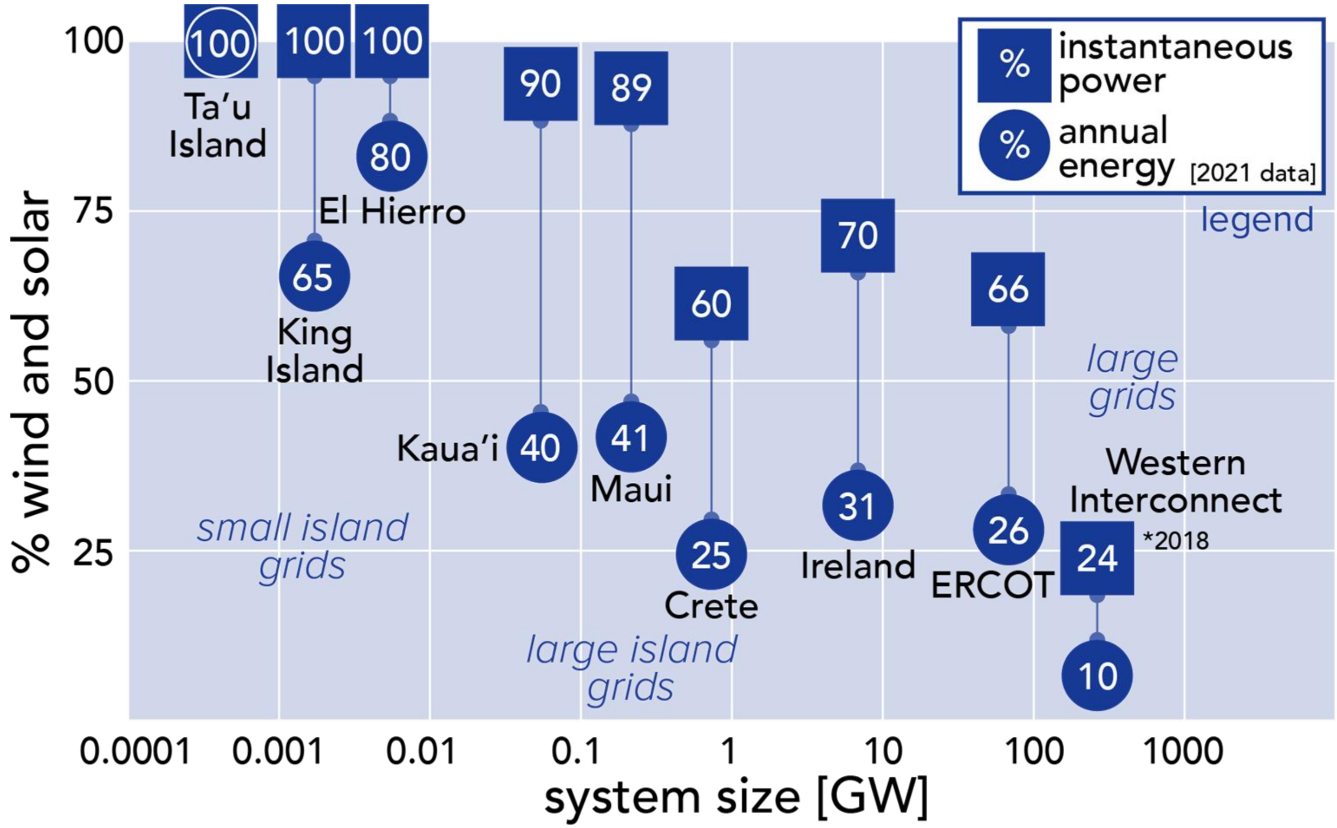
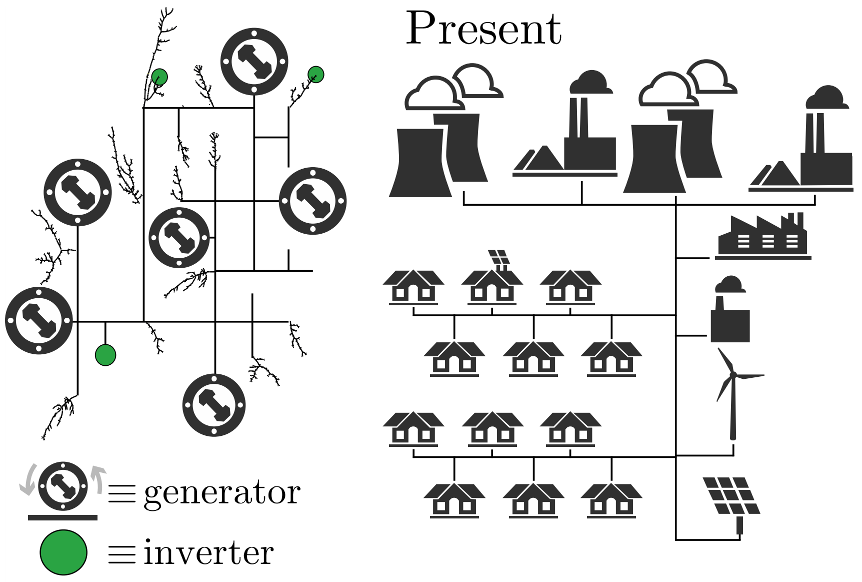
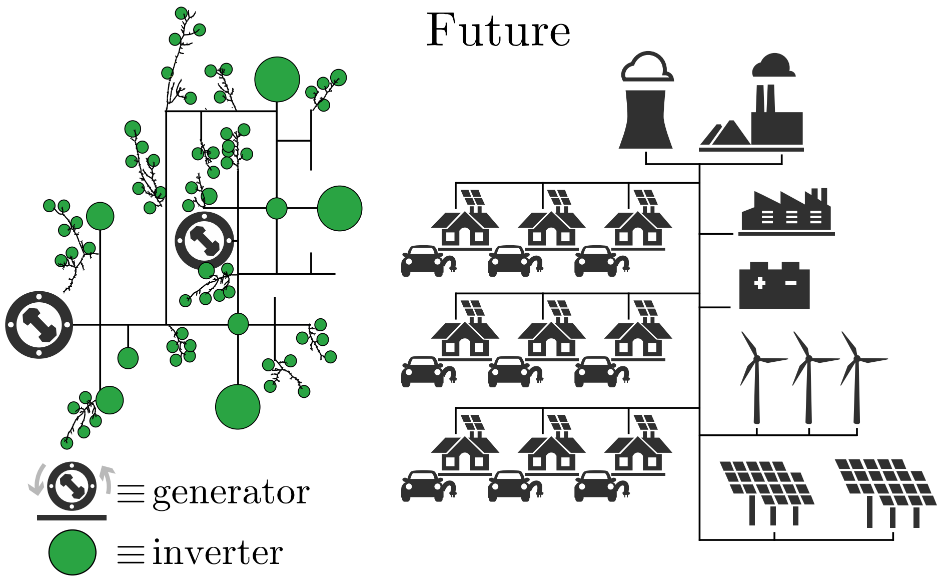
Renewable Utilization for Various Grid Sizes



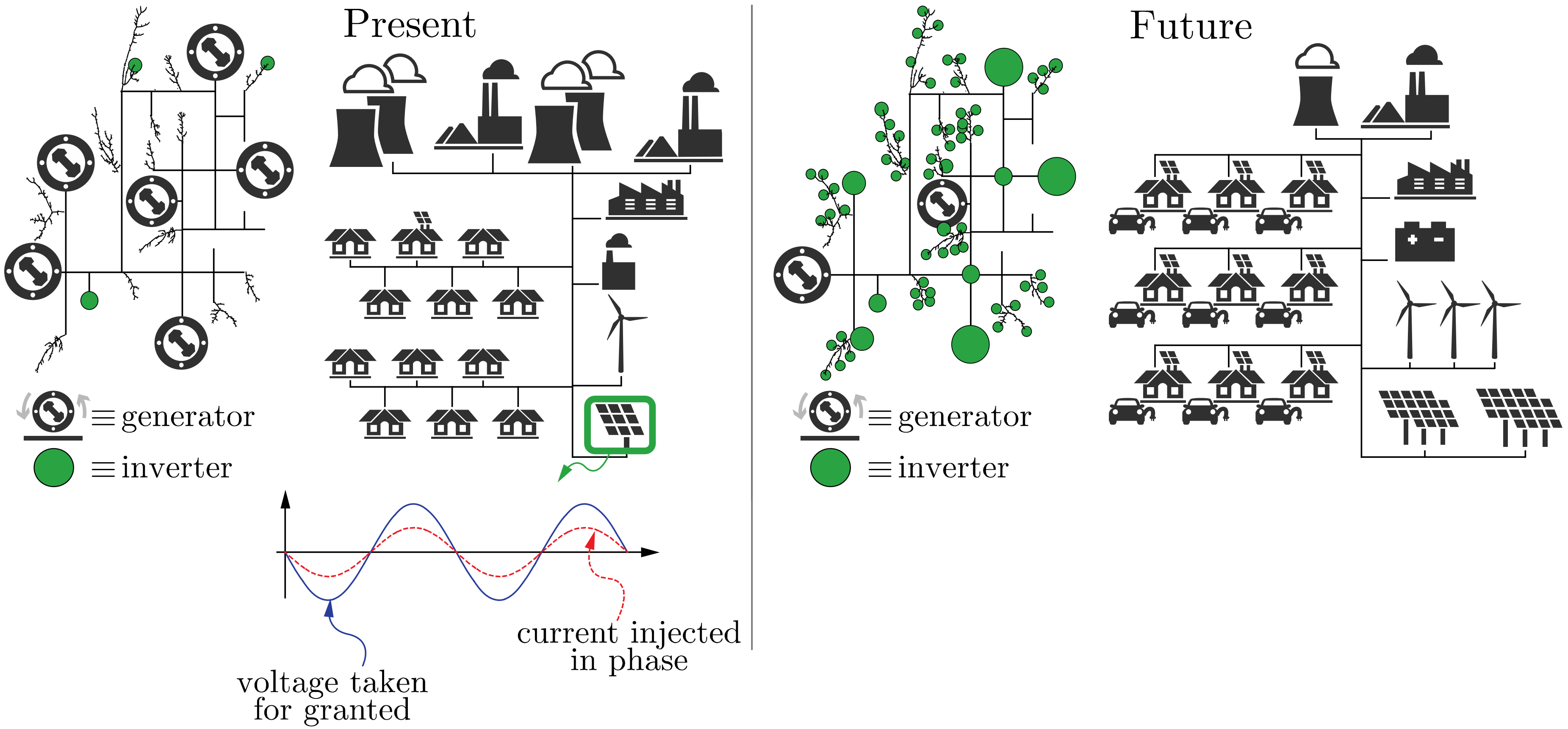
- Engineering challenges grow with system size and complexity
- Need scalable, robust, and resilient methods for system operation

A Vision for the Future Grid

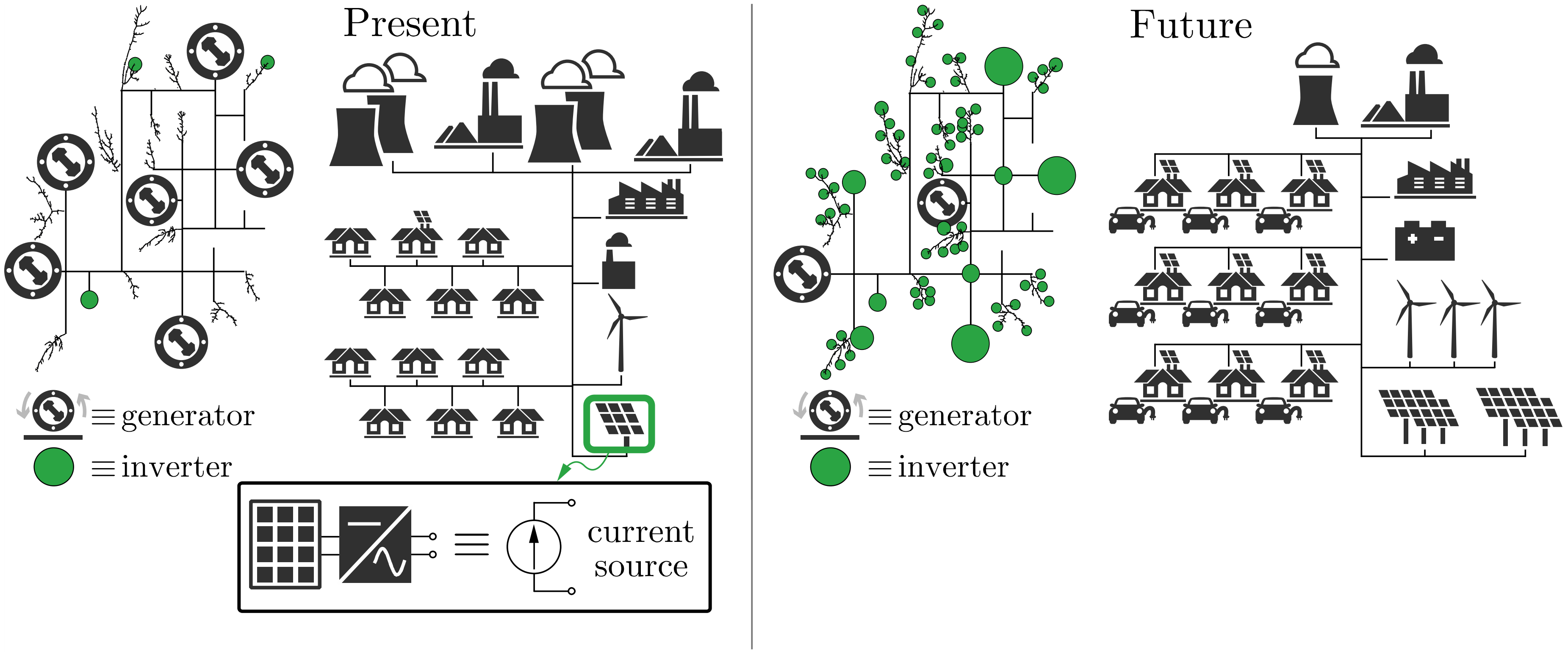




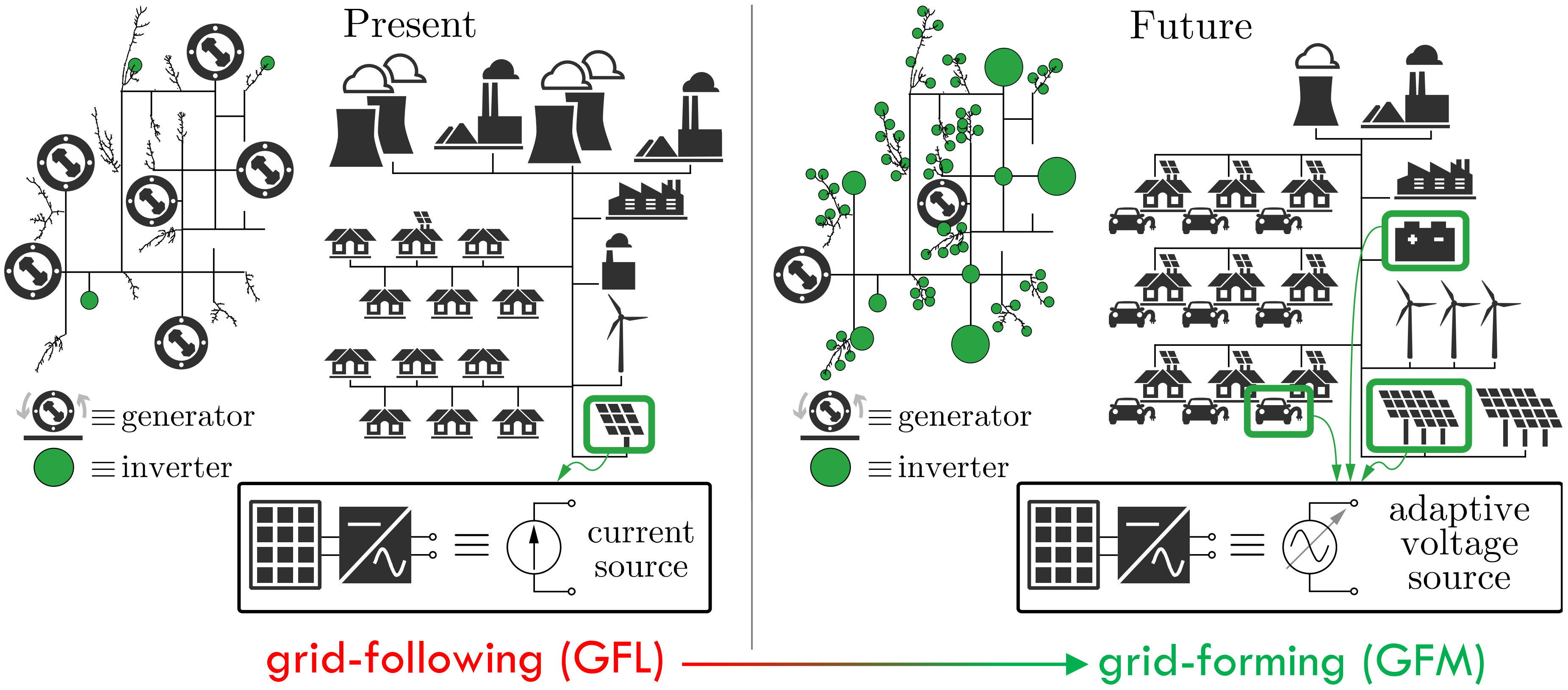
A Vision for the Future Grid



A Vision for the Future Grid



A Vision for the Future Grid

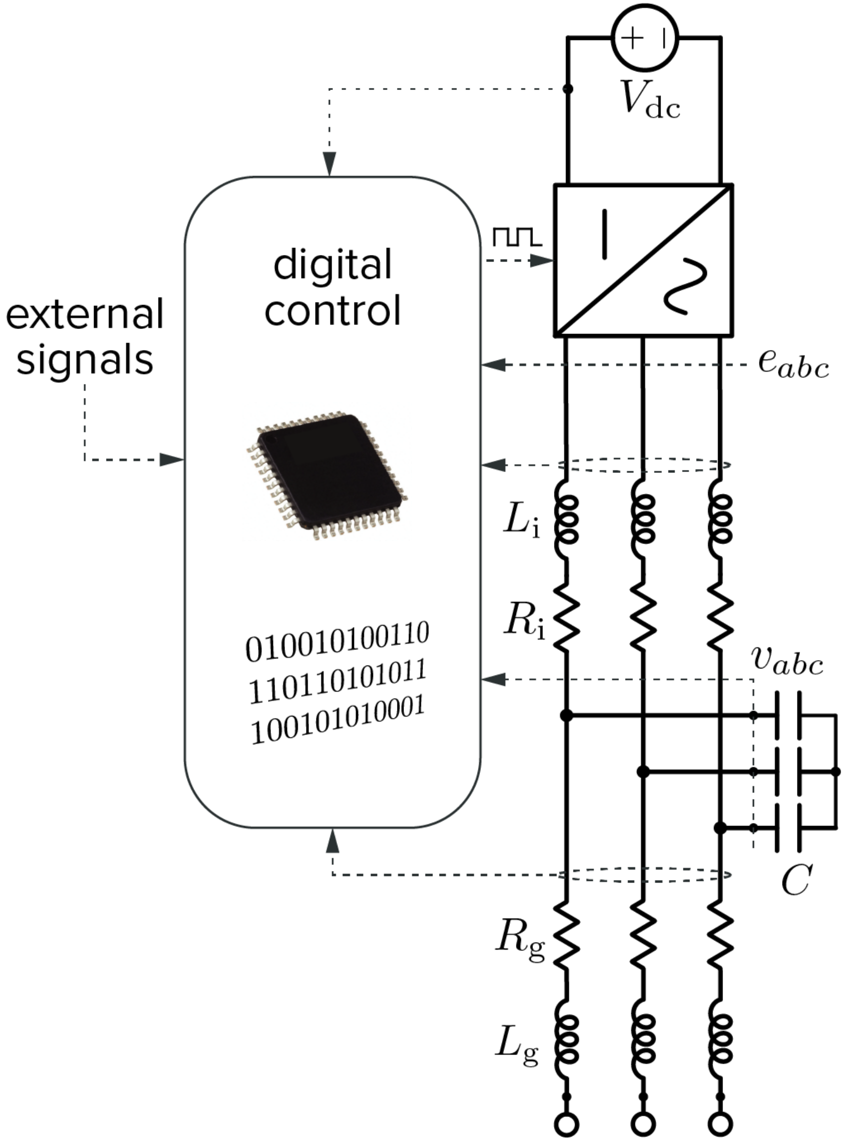


Achieving 2 goals simultaneously:

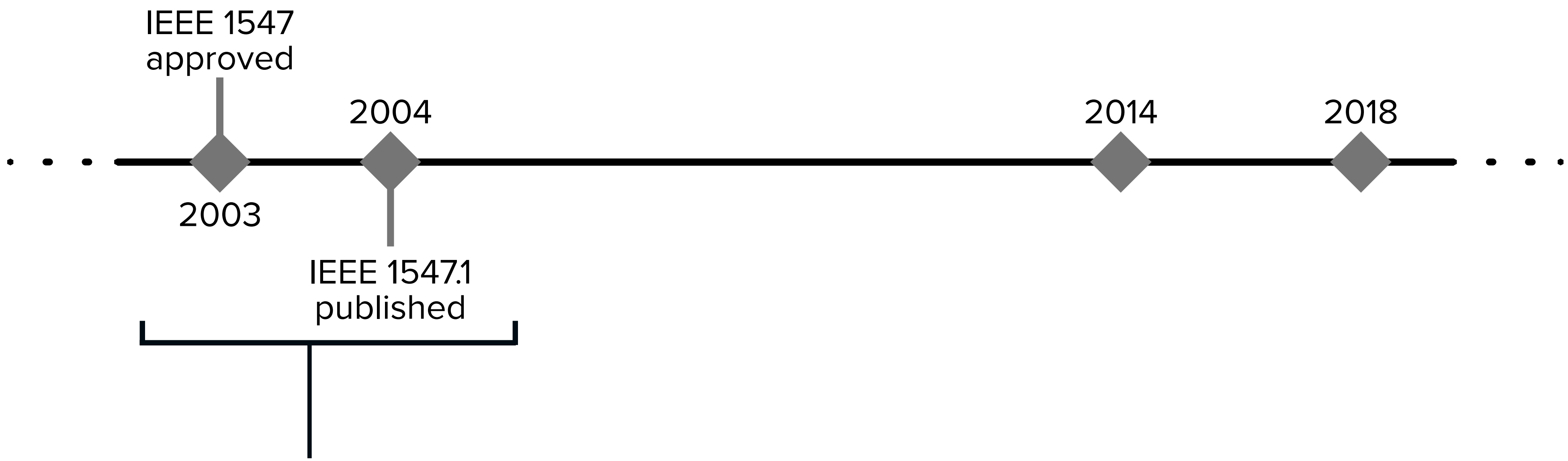
- Break down barriers that limit adoption of renewable energy
- Realize a bottom-up system that works resiliently at any scale

Desired Characteristics of a GFM Inverter

- Can operate in standalone or with many other GFMs
- Proportional power sharing among units
- Works when connected to a stiff or weak grids
- Automatic synchronization
- Exogenous signals can adjust power delivery
- Has current limiter to prevent overcurrents
- Integrates with dc-side controls



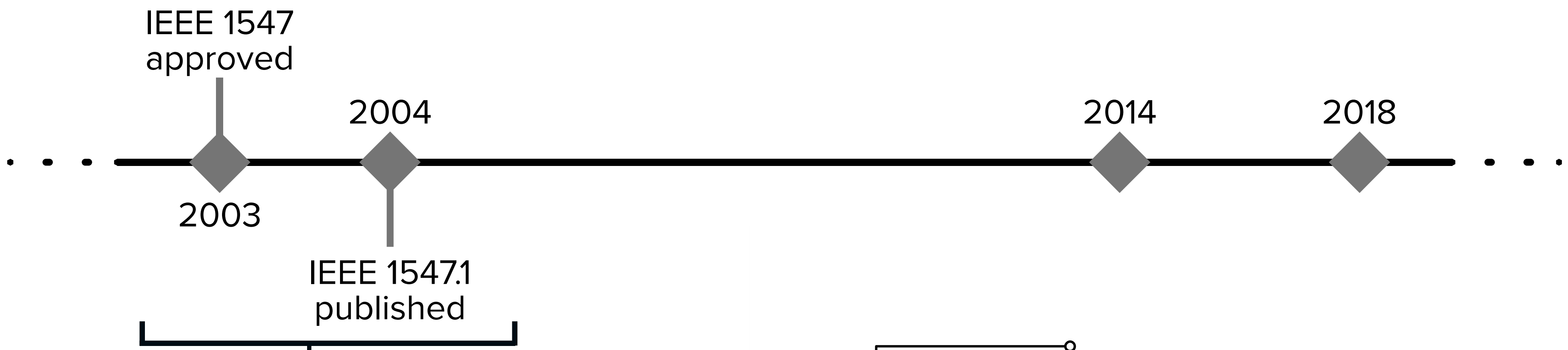
An Evolving Regulatory Landscape



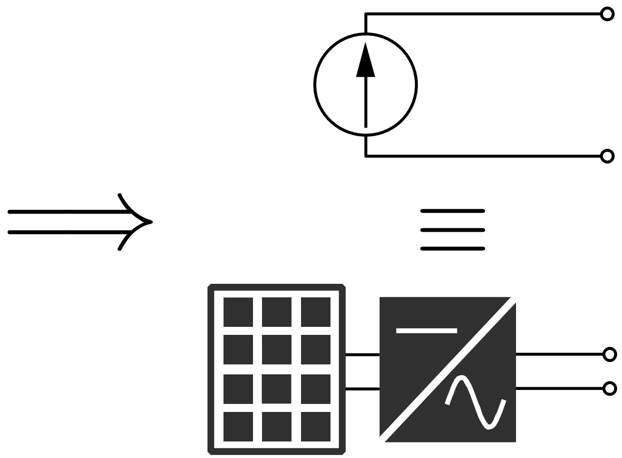
The inverter shall:

- NOT regulate voltage
- TRIP under abnormal voltage/frequency

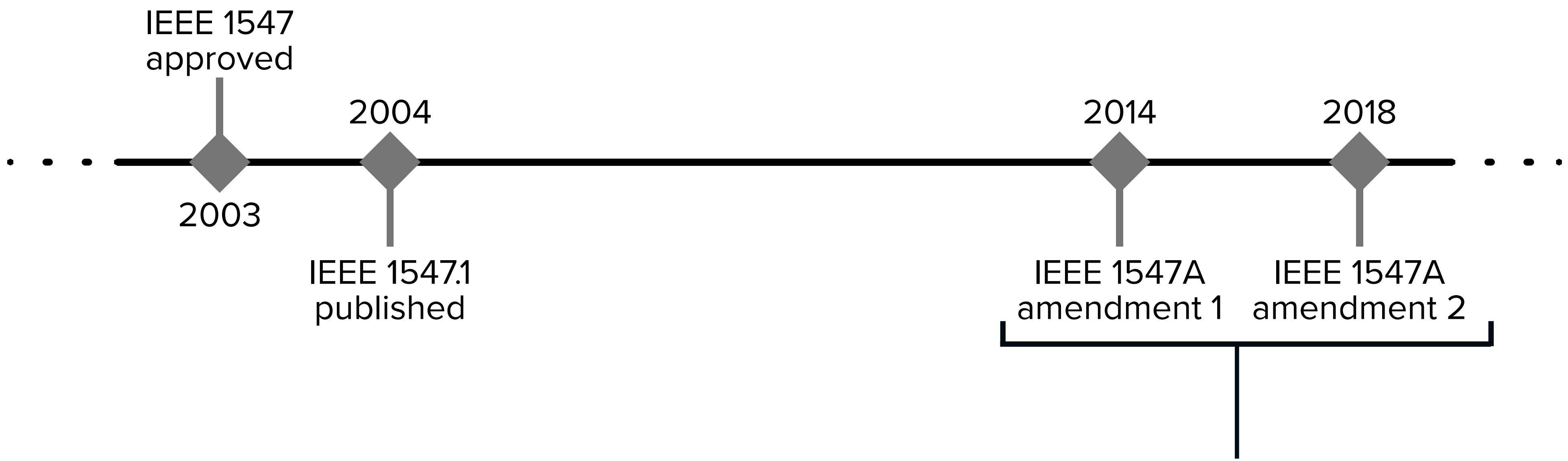
An Evolving Regulatory Landscape



GFL unit acts as passive current injection & disconnects abruptly



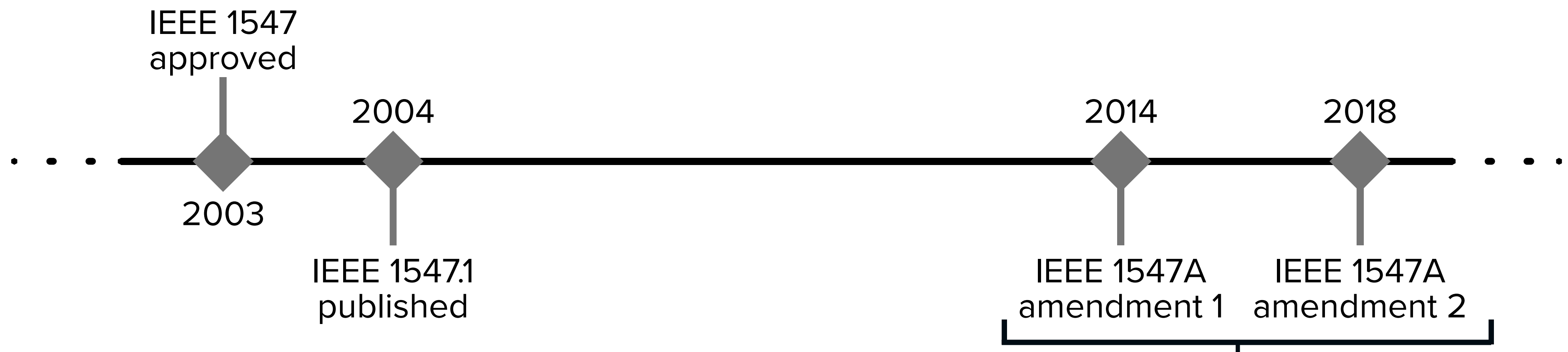
An Evolving Regulatory Landscape

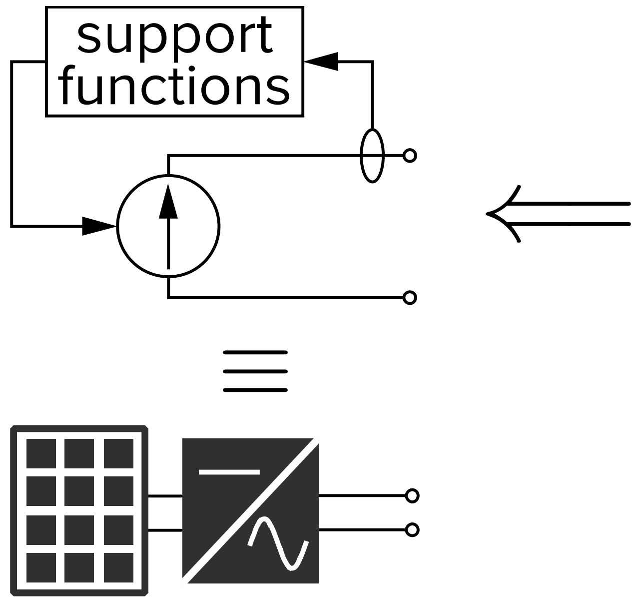


The inverter shall:

- Assist voltage regulation
- Ride-through abnormal voltage/frequency

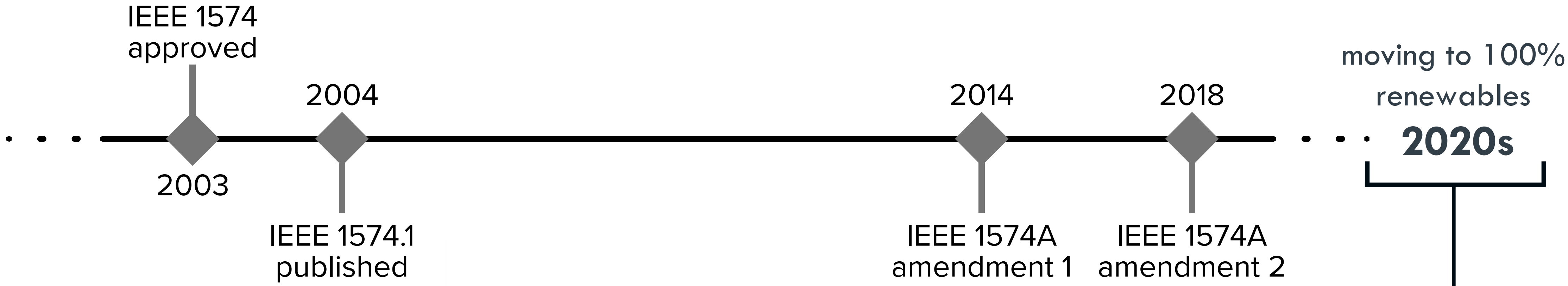
An Evolving Regulatory Landscape

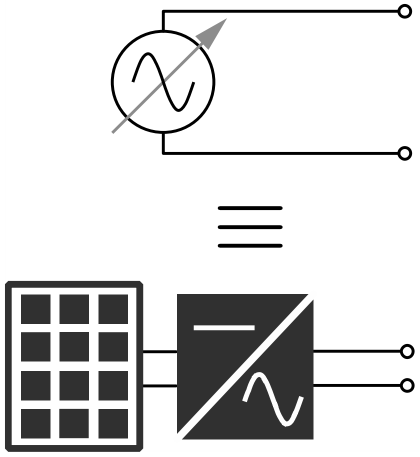




GFL supplemented with functions for grid support

An Evolving Regulatory Landscape





Full-fledged grid-forming (GFM) functionality needed to realize ambitious renewable targets[1]

[1] List of territories, states, cities, communities committed to 100% renewable energy: www.sierraclub.org/ready-for-100

Remaking the Electric Power Industry

The organization that will bring GFM technologies onto grids:

- Duration: 2022-2027
- Co-Leads: UT, NREL, EPRI
- Total Funds: $34.9M
- Federal Funds: $25M
- Membership:
  - 12 universities
  - 4 national labs
  - 25 industry members

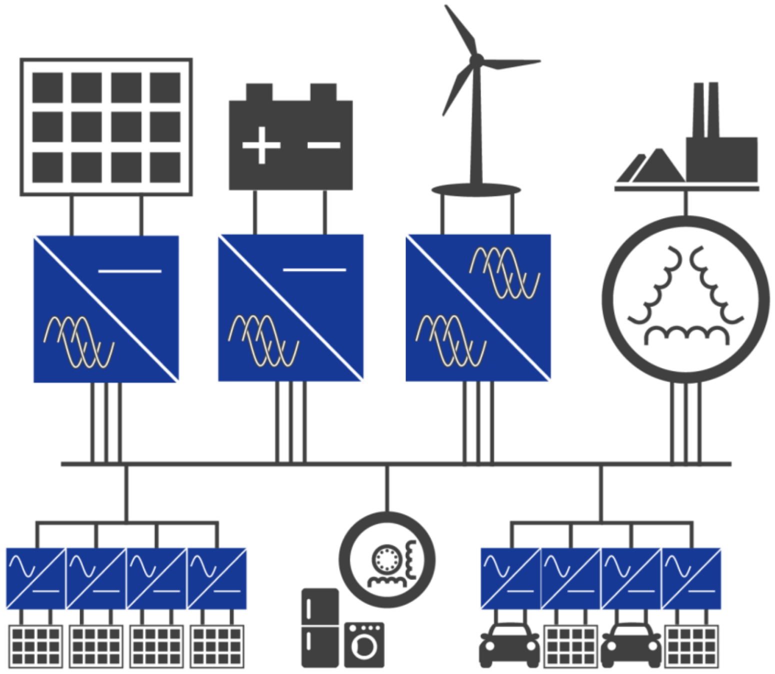
unifi consortium

universal interoperability for grid-forming inverters

Funded by:

SOLAR ENERGY TECHNOLOGIES OFFICE
U.S. Department Of Energy

Unifying Technologies Across All Scales



unifi will unify

- New & Old (GFM with GFL+Machines)
- Local & Global (controls)
- Slow & Fast (timescales of operation)
- Big & Small (inverters to aggregations)
- Solar, Wind, & Storage (technologies)

A Team with Crosscutting Perspectives Across the Electric Power Industry

National Labs & Research Institutes

NREL Transforming ENERGY
EPRI
Berkeley Lab
Sandia National Laboratories
PNNL

Universities

The University of Texas at Austin
NC State University
ASU
GT
Cal
I
P
UAF
VT

unifi consortium

Industry

Siemens
Danfoss
SMA
Eaton
Analog Devices
Ørsted
Hitachi Energy
OPAL-RT Technologies
Typhoon HIL

Utilities & System Operators

Southern California Edison
Hawaiian Electric
PG&E
Xcel Energy
MISO
Southern Company
New York Power Authority
ComEd
PGE
CAISO
PacifiCorp
ISO new england

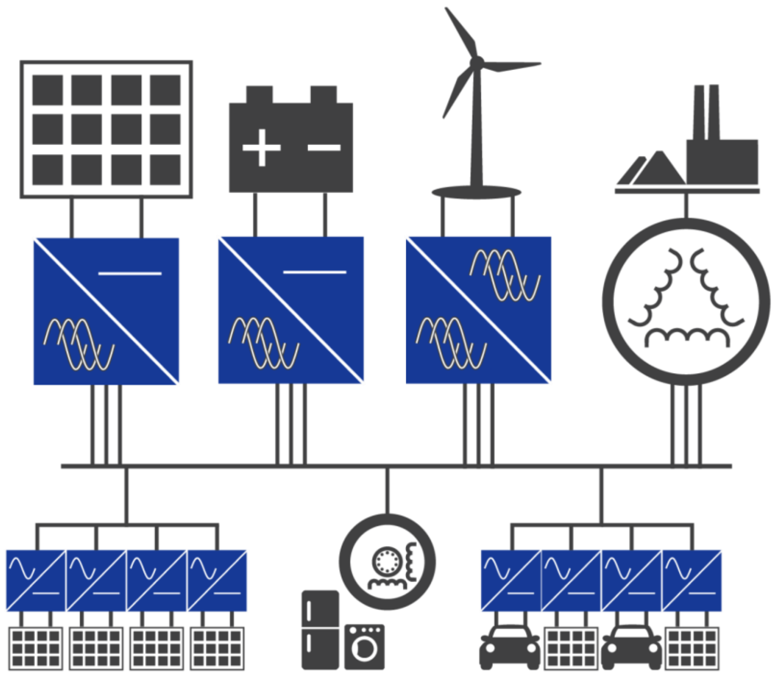
A Team with Crosscutting Perspectives Across the Electric Power Industry

These + many others will need to work towards consensus

Universal Interoperable Operation is Essential

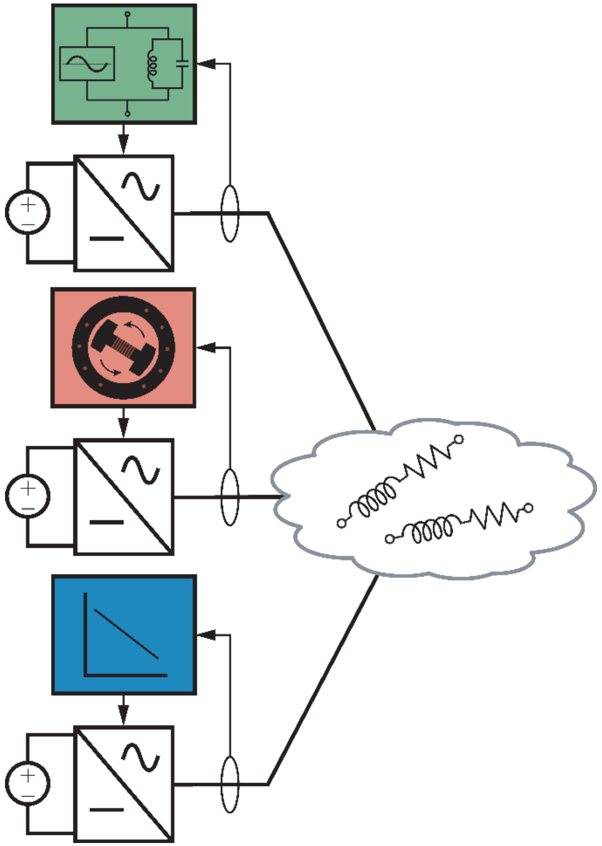
Seeking inspiration from Bluetooth®

& cultivating an ecosystem of innovation

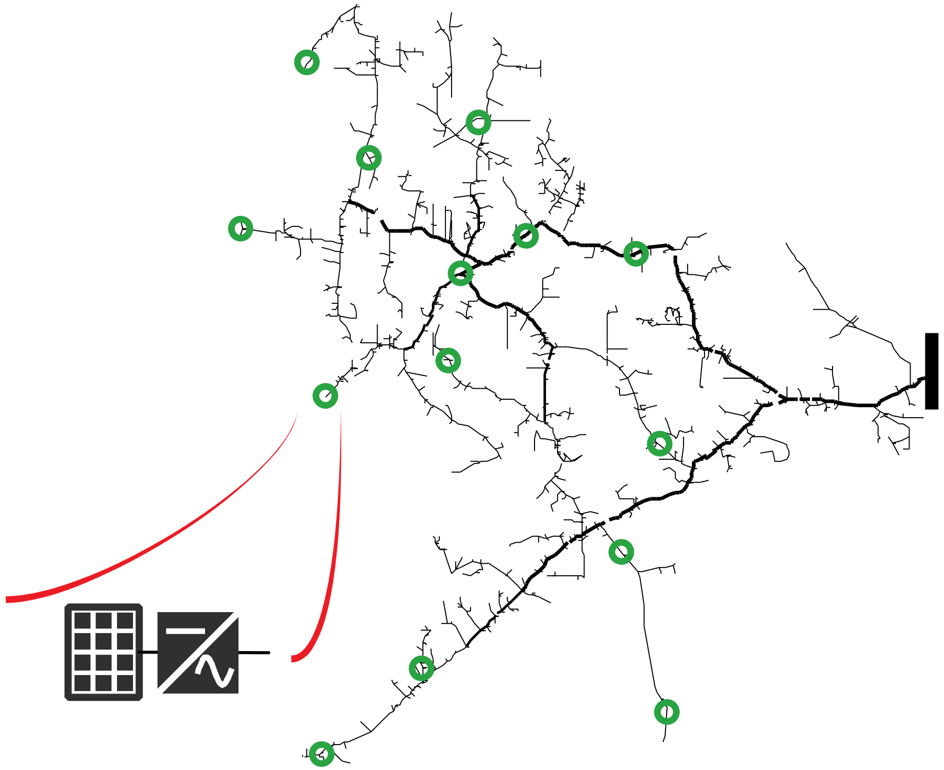


Interoperable GFM controls will

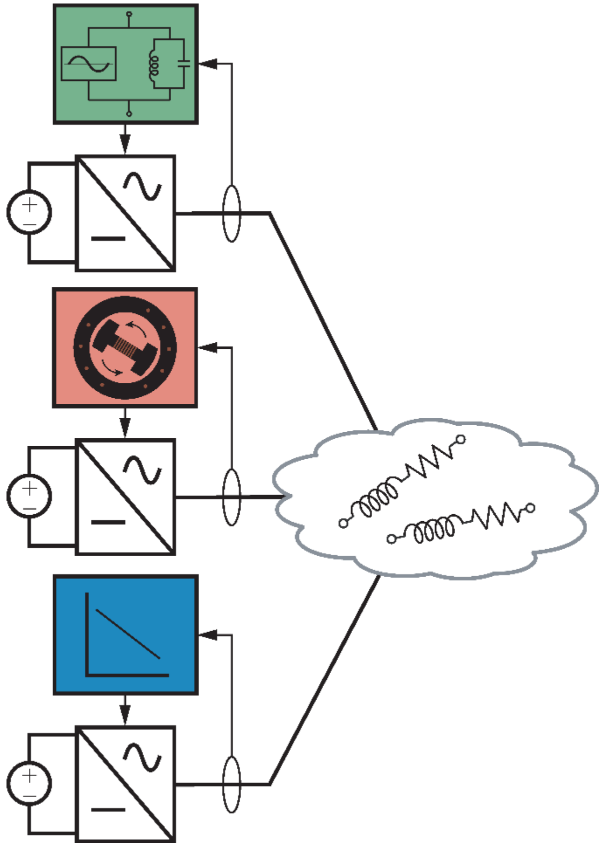
- Enable all manufacturers to innovate
- Coexist with proprietary functions
- Streamline first-principles-based standards
- Be black-box testable for certification



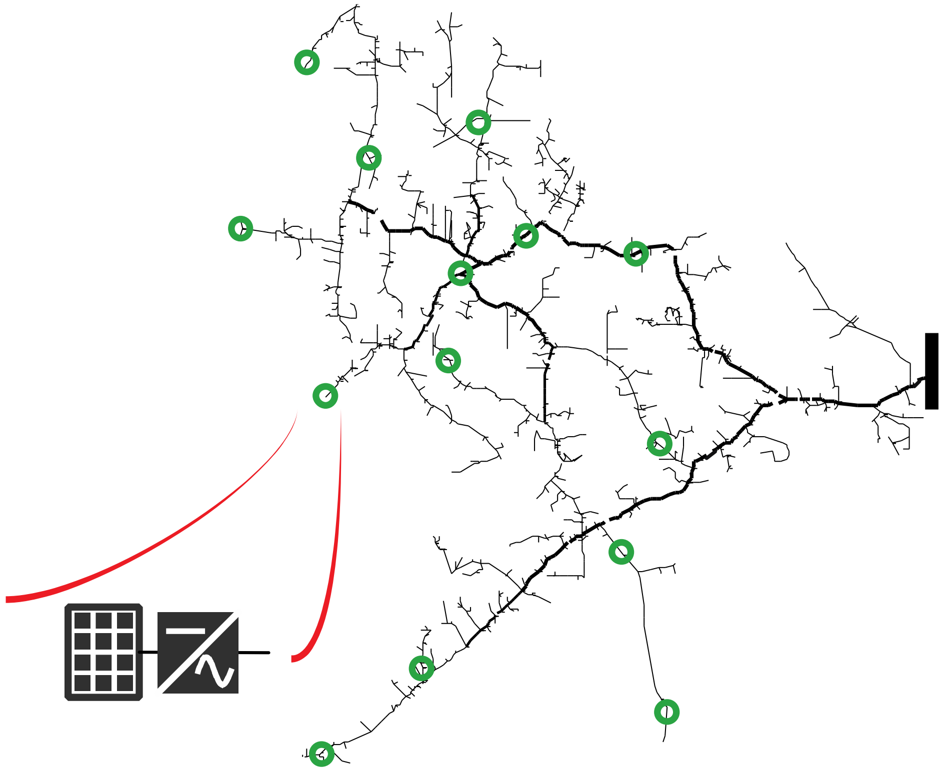
Interoperability



Self-organizing GFM Networks

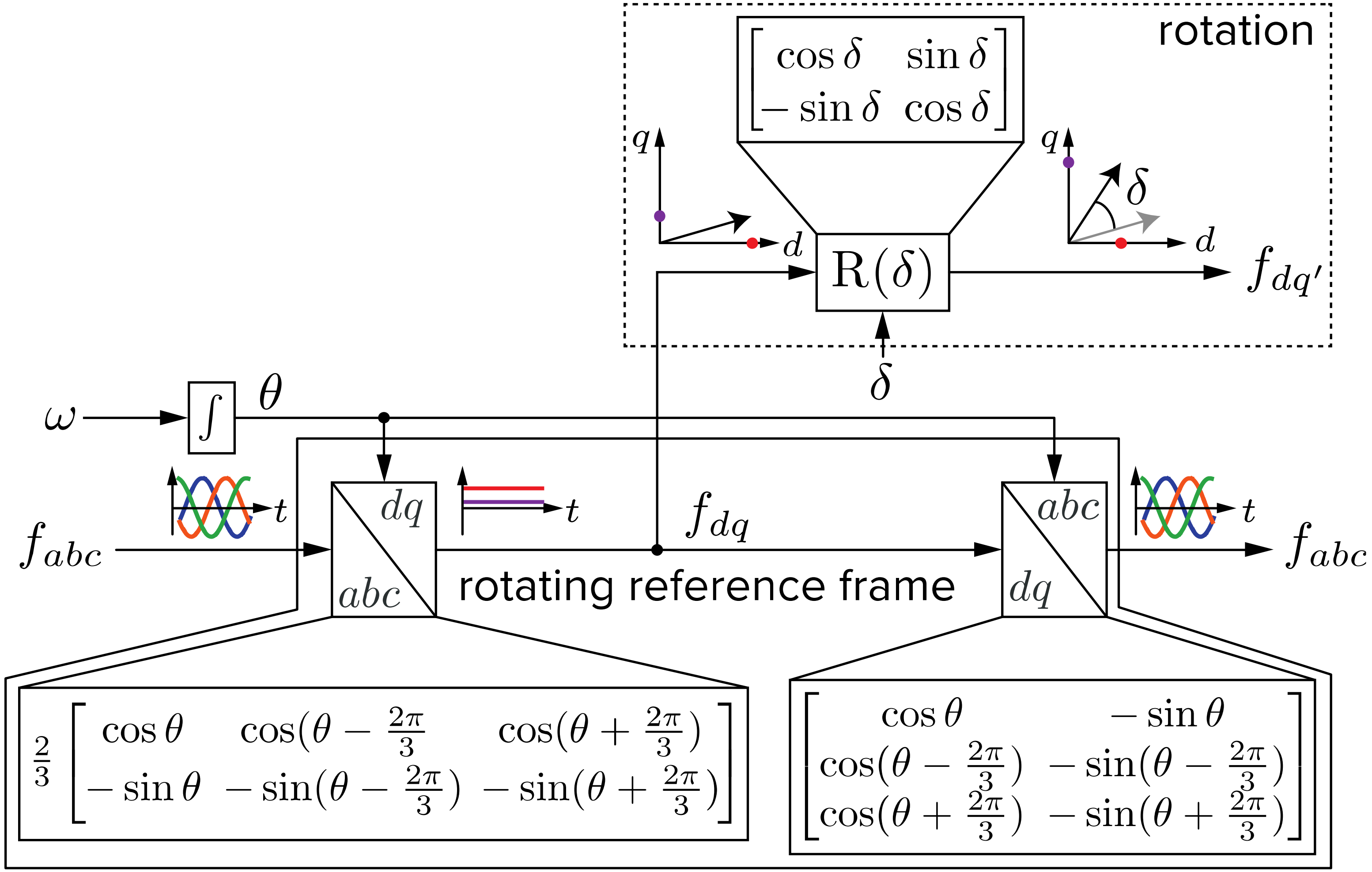


Interoperability



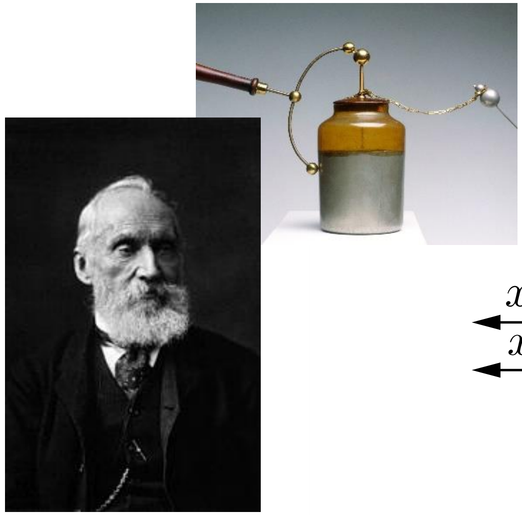
Self-organizing GFM Networks

Reference Frame Basics for Three-phase Systems



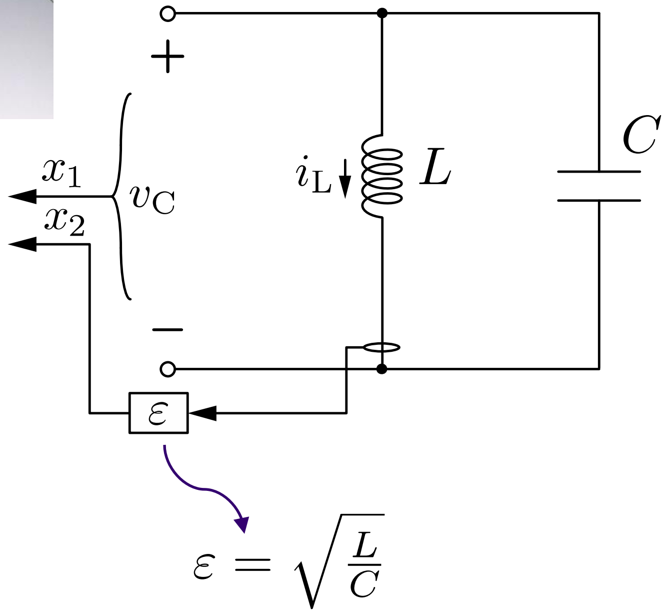
We Begin with a Humble Little Circuit

The simple harmonic oscillator lies at the heart of modern GFM control

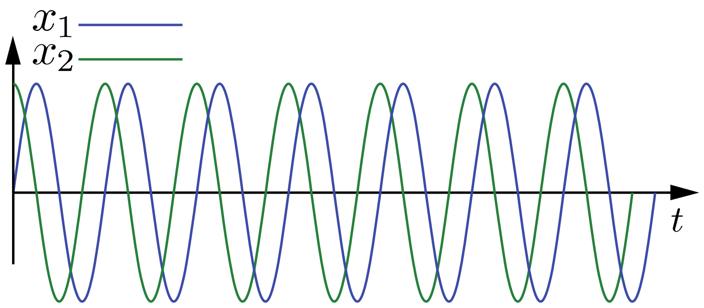


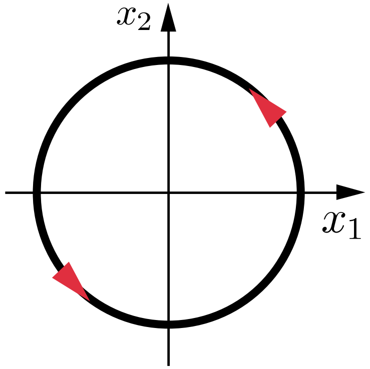
William Thomson

1824-1907



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$





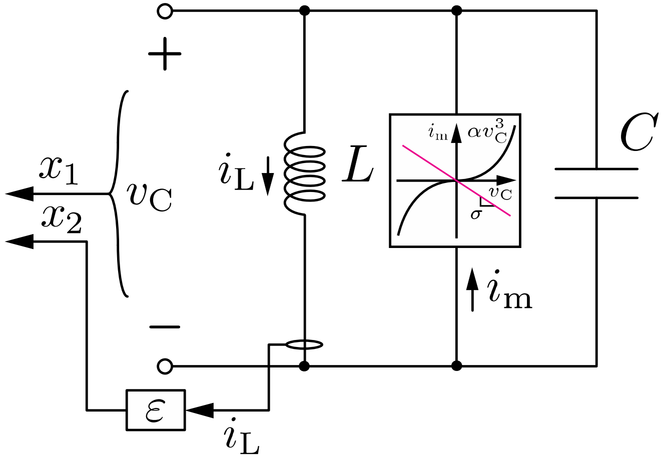
[1] Blanchard, "The history of electrical resonance," Bell System Technical Journal, 1941.

Early Oscillator Models with Self-regulating Limit Cycles

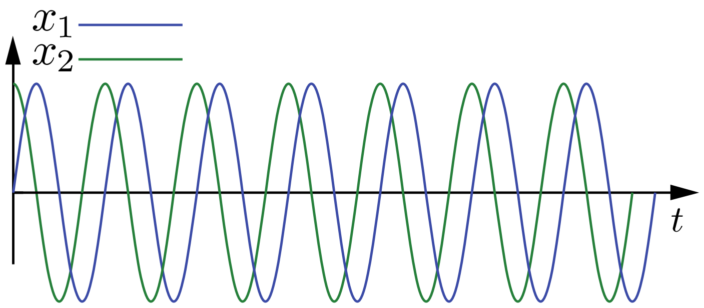
The van der Pol oscillator is a stepping-stone towards a power application

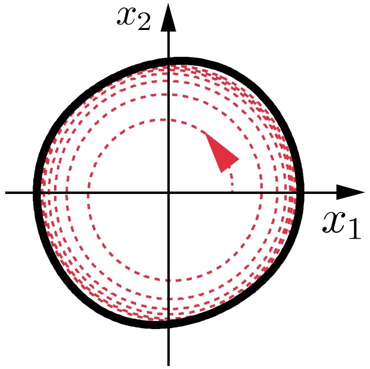
Balthasar van der Pol

1889-1959



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} \overbrace{\boxed{\varepsilon\omega_0(\sigma-\alpha x_1^2)}}^{i_m/C} & -\omega_0 \\ \omega_0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$





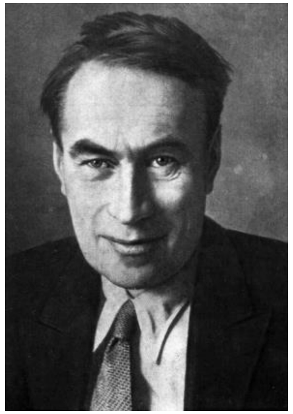
[1] van der Pol, "On "relaxation-oscillations”," The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 1926.

[2] B. Johnson, Dhople, Hamadeh, Krein, “Synchronization of parallel single-phase inverters using virtual oscillator control,” TPEL, 2014.

[3] Awal, Yu, Tu, Lukic, Husain, "Hierarchical control for virtual oscillator based grid-connected and islanded microgrids," TPEL, 2020.

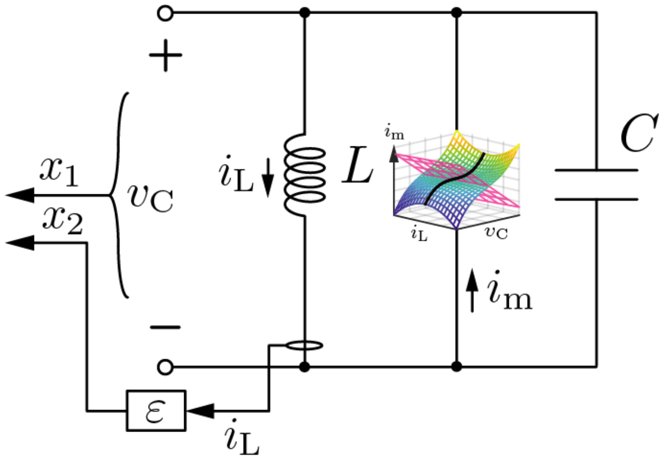
Early Oscillator Models with Self-regulating Limit Cycles

Andronov's oscillator gives harmonic-free waveforms useful for high power quality

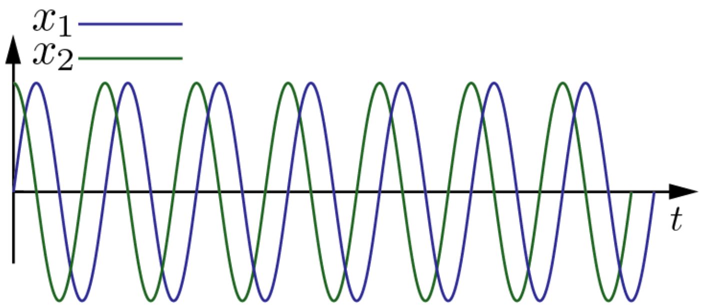


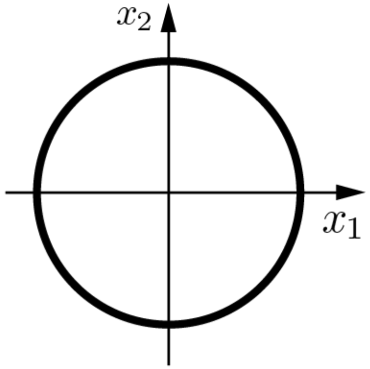
Aleksandr Andronov

1901-1952



$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} \overbrace{\boxed{\xi(2X_{\mathrm{nom}}^2 - \|x\|^2)}}^{i_{\mathrm{m}}/C} & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$





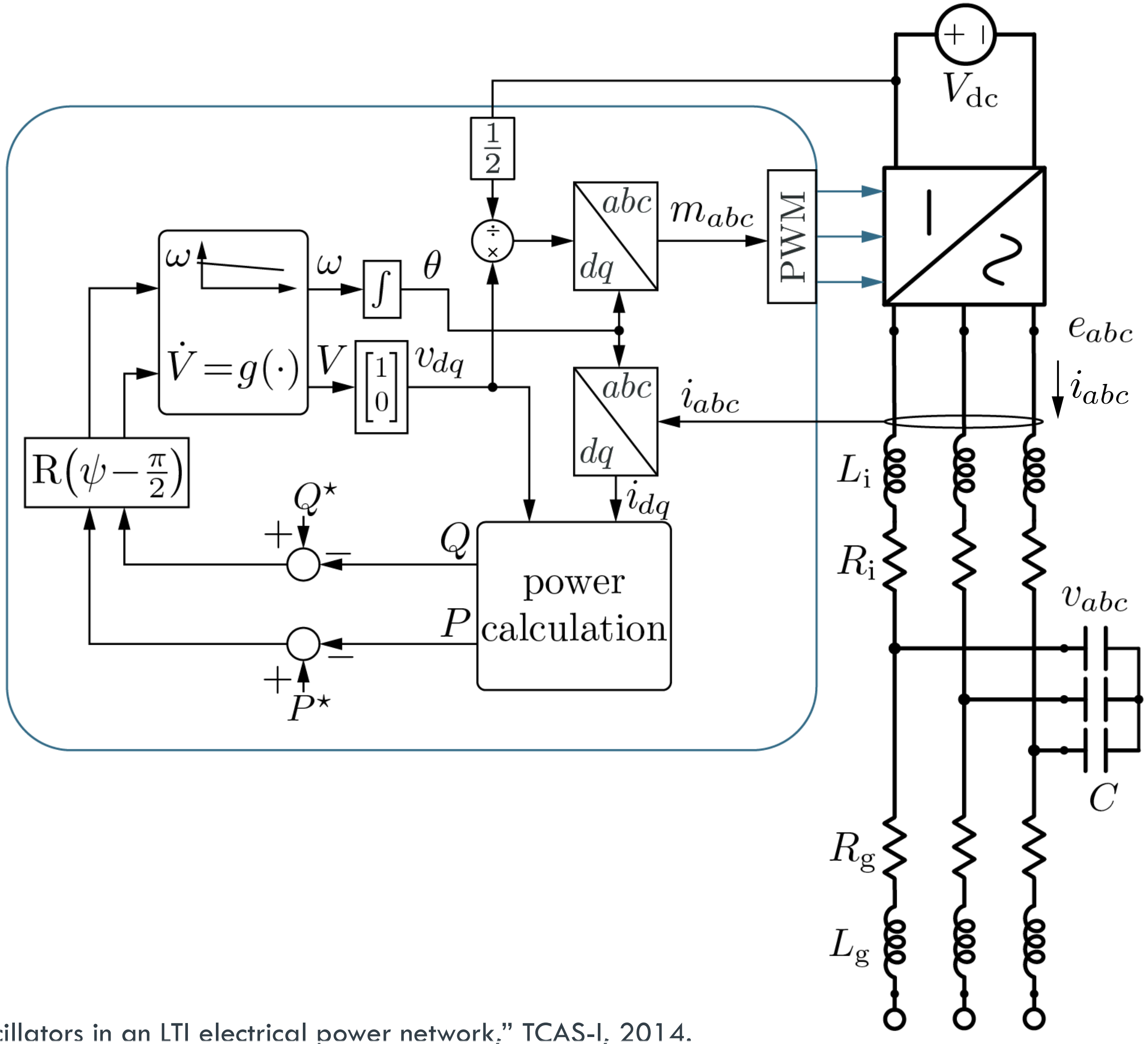
[1] Andronov, Leontovich, Gordon, Maier, "Theory of Bifurcations of Dynamical Systems on a Plane," Israel Program Science Translation, 1971.

[2] Columbino, Gross, Dorfler, "Global phase and voltage synchronization for power inverters: A decentralized consensus-inspired approach," CDC, 2017.

[3] Lu, Dutta, Purba, Dhople, Johnson, "A grid-compatible virtual oscillator controller: Analysis and design" ECCE, 2019.

Approach #1: Dispatchable Virtual Oscillator Control

- Transformed oscillator gives model here
- Newest GFM type in existence



[1] Johnson, Dhople, Hamadeh, Krein, "Synchronization of nonlinear oscillators in an LTI electrical power network," TCAS-I, 2014.

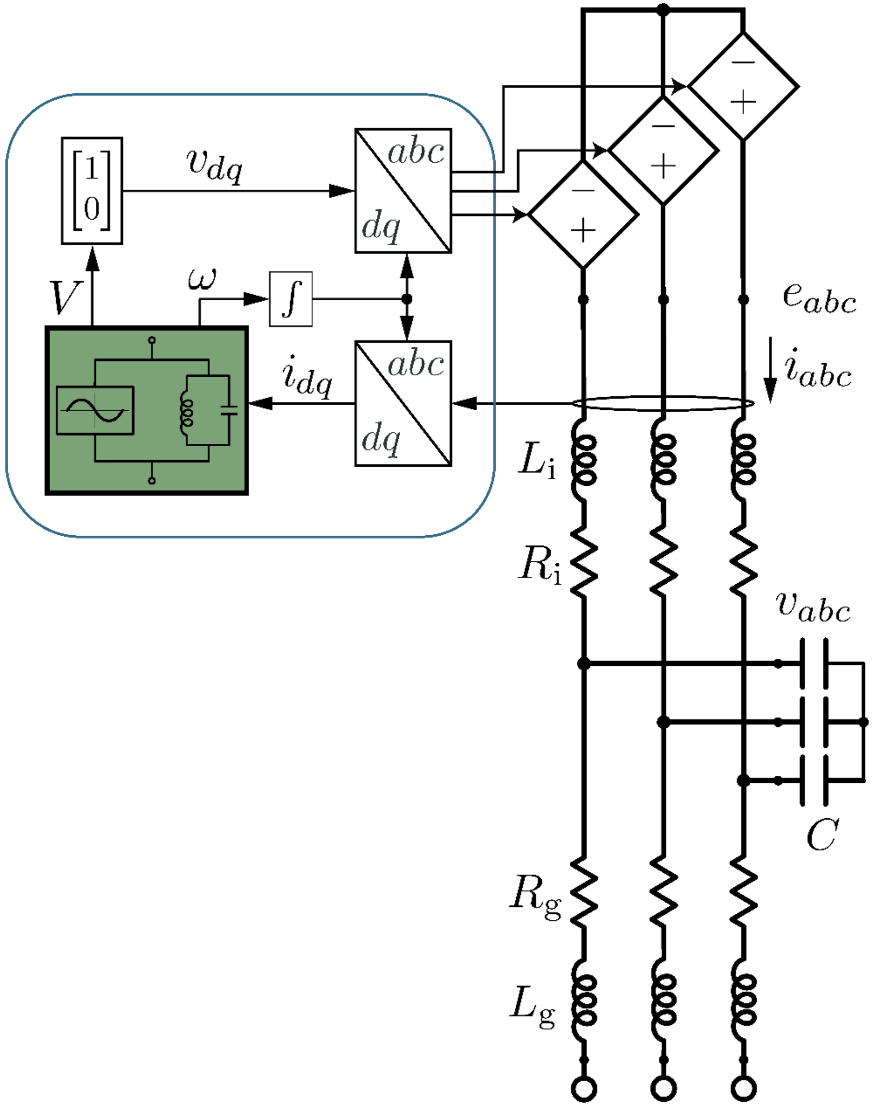
[2] Colombino, Groß, Dorfler, "Global phase and voltage synchronization for power inverters: A decentralized consensus-inspired approach," CDC, 2017.

Approach #1: Dispatchable Virtual Oscillator Control

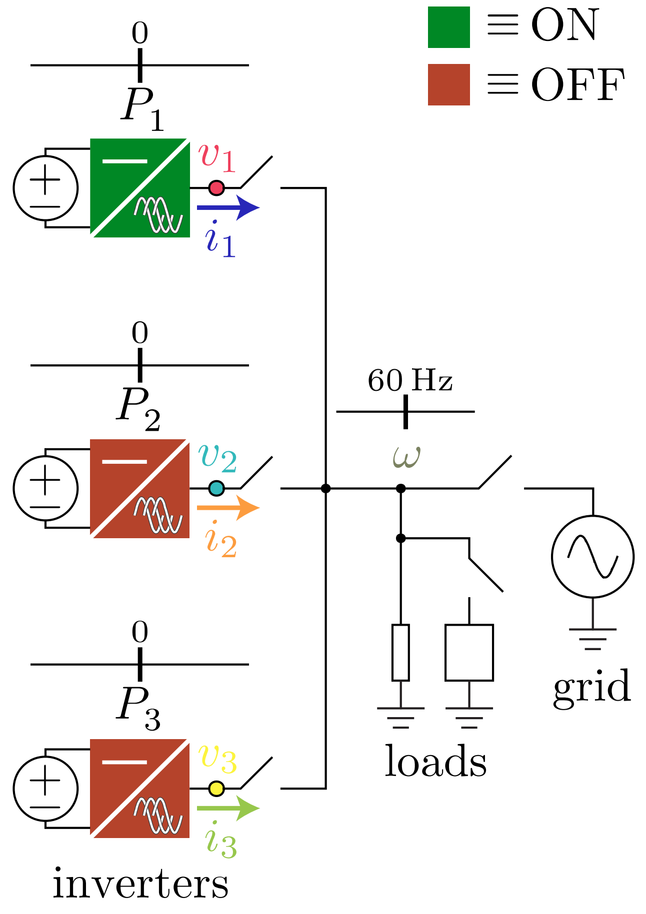
Control equations are given by

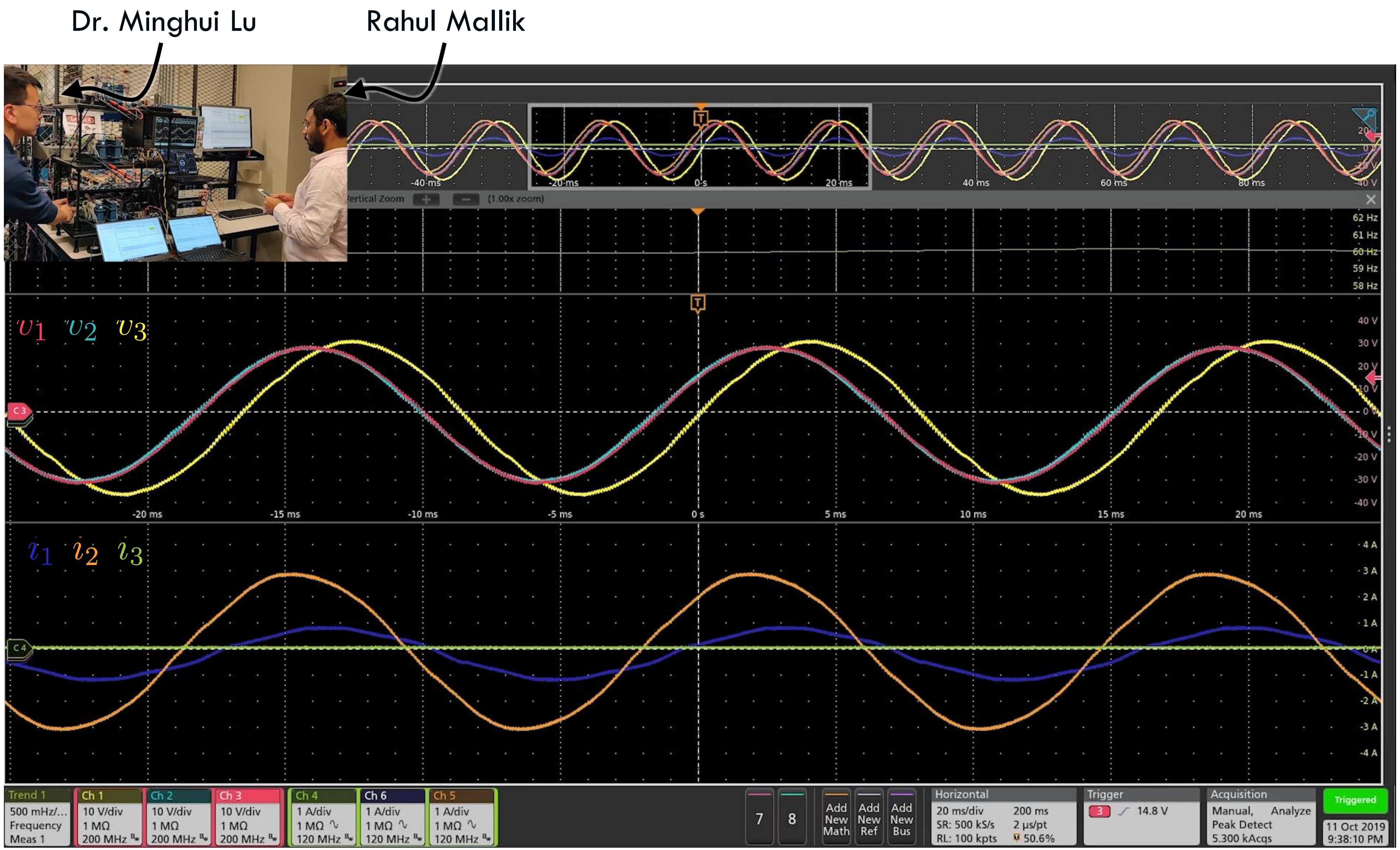
$$\omega = \omega_0 + \frac{\omega_0 \kappa_1}{V^2} \begin{bmatrix} 1 & 0 \end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P \\ Q^\star - Q \end{bmatrix},$$

$$\dot{V} = \omega_0 \kappa_2 V (V_0^2 - V^2) + \frac{\omega_0 \kappa_1}{V} \begin{bmatrix} 0 & 1 \end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P \\ Q^\star - Q \end{bmatrix}.$$



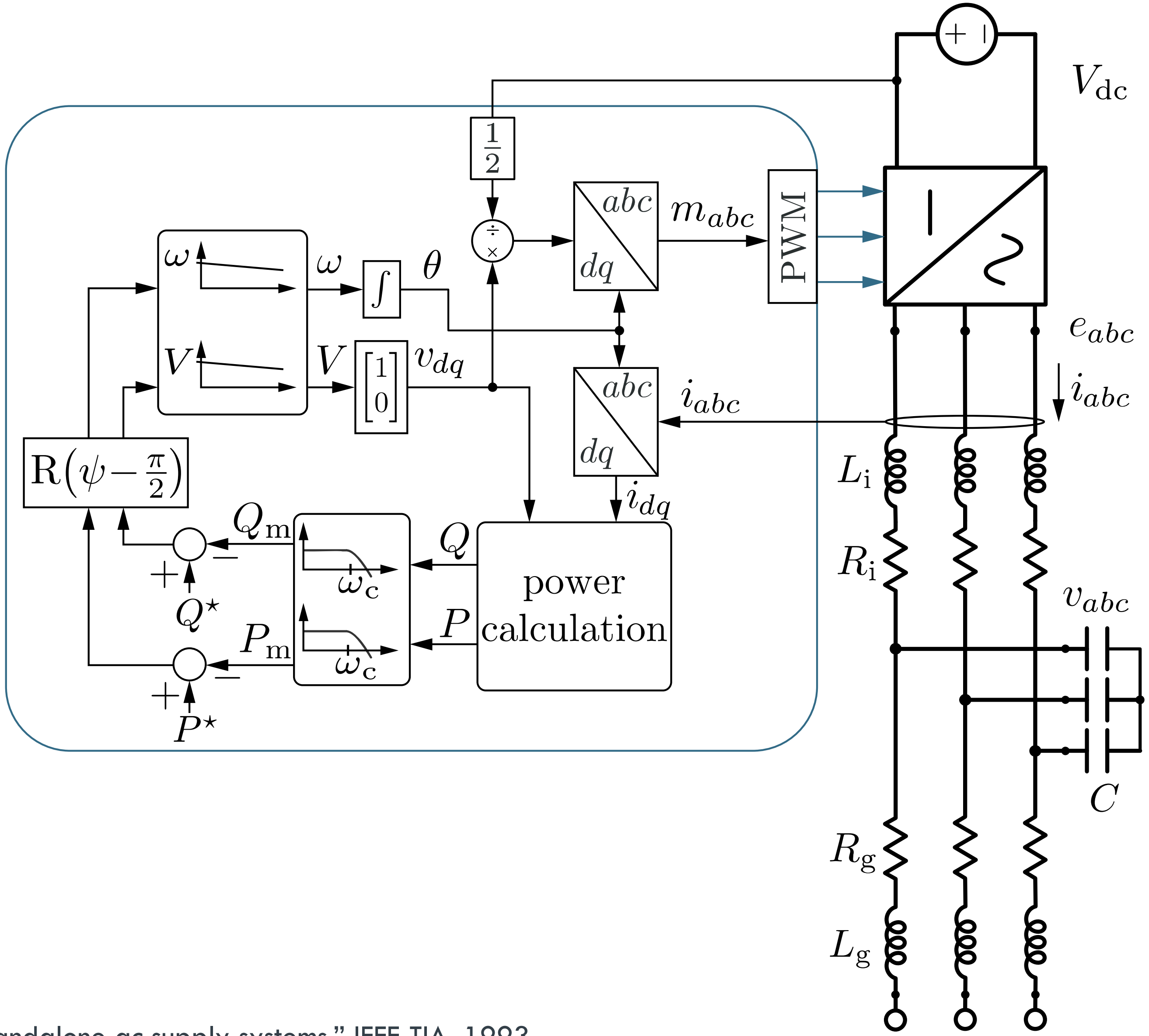
Illustrating Versatile Performance on Hardware with dVOC Control





Approach #2: Droop Control

- Oldest GFM method in existence
- Inspired by machine droop laws



[1] M. Chandorkar, et. al, “Control of parallel connected inverters in standalone ac supply systems,” IEEE TIA, 1993

[2] J. Guerrero, et. al, “Hierarchical control of droop-controlled dc and ac microgrids,” IEEE TIE, 2011.

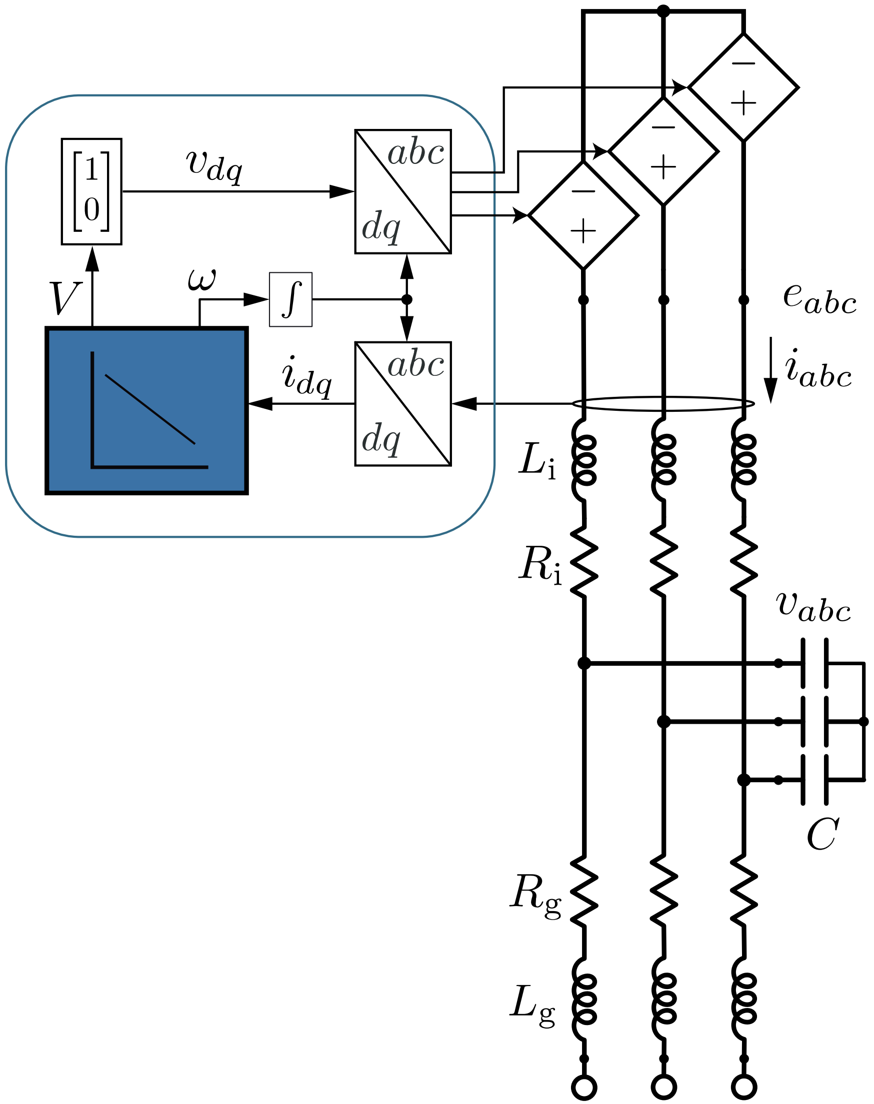
Approach #2: Droop Control

Control equations are given by

$$\omega = \omega_0 + \tfrac{1}{d_\mathrm{f}} \begin{bmatrix}1 & 0\end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P_\mathrm{m} \\ Q^\star - Q_\mathrm{m} \end{bmatrix},$$

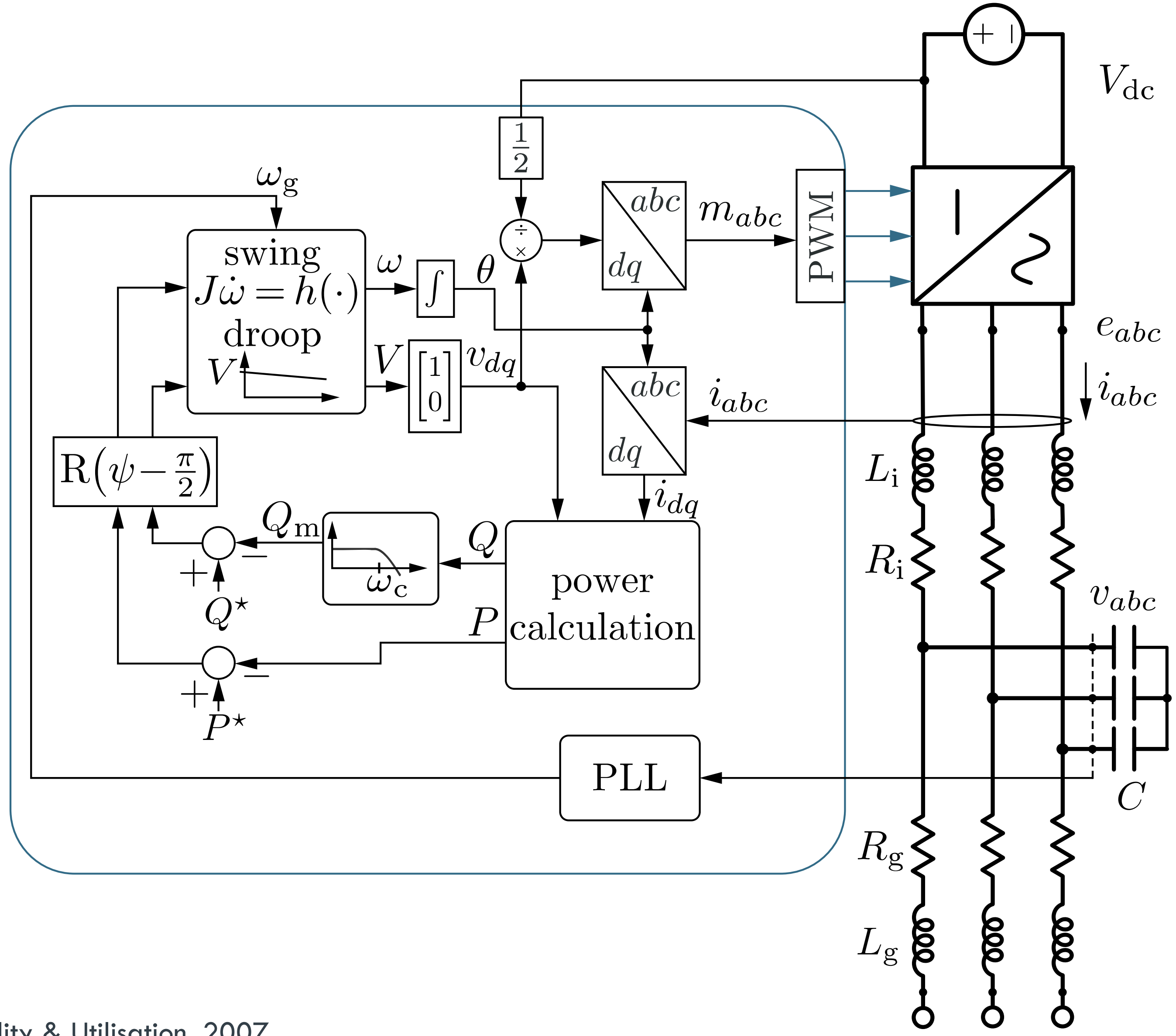
$$V = V_0 + \tfrac{1}{d_\mathrm{v}} \begin{bmatrix}0 & 1\end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P_\mathrm{m} \\ Q^\star - Q_\mathrm{m} \end{bmatrix},$$

$$\frac{1}{\omega_\mathrm{c}} \begin{bmatrix} \dot{P}_\mathrm{m} \\ \dot{Q}_\mathrm{m} \end{bmatrix} = - \begin{bmatrix} P_\mathrm{m} \\ Q_\mathrm{m} \end{bmatrix} + \begin{bmatrix} P \\ Q \end{bmatrix}.$$



Approach #3: Virtual Synchronous Machine Control

- Emulate machine dynamics digitally
- Popular due to familiar behavior



[1] Beck, Hesse, "Virtual synchronous machine," Electrical Power Quality & Utilisation, 2007.

[2] D'Arco, Suul, Fosso, "A virtual synchronous machine implementation for distributed control of power converters in smartgrids," EPSR, 2014.

Approach #3: Virtual Synchronous Machine Control

Control equations are given by

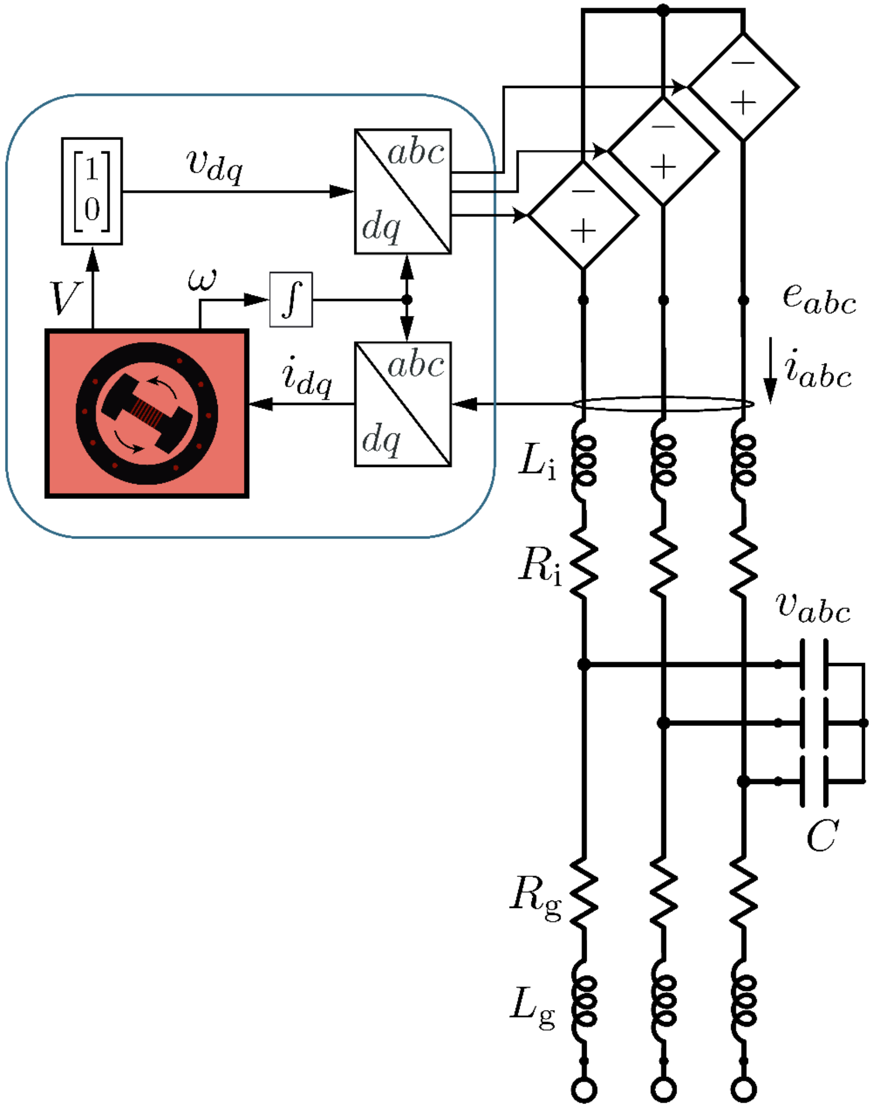
$$J\dot{\omega} = -\omega + \omega_0 + \frac{d_\mathrm{d}}{d_\mathrm{f}}(\omega_\mathrm{g} - \omega) + \frac{1}{d_\mathrm{f}}\begin{bmatrix}1 & 0\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}P^\star - P\\ Q^\star - Q_\mathrm{m}\end{bmatrix},$$

$$V = V_0 + \frac{1}{d_\mathrm{v}}\begin{bmatrix}0 & 1\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}P^\star - P\\ Q^\star - Q_\mathrm{m}\end{bmatrix},$$

$$\frac{1}{\omega_0}\dot{\eta} = \begin{bmatrix}0 & 1\end{bmatrix}\mathrm{R}(\alpha)\mathrm{R}(\delta)\mathrm{T}(\omega_0 t)v,$$

$$\frac{1}{\omega_0}\dot{\alpha} = \frac{k_\mathrm{P}}{\omega_0}\dot{\eta} + k_\mathrm{I}\eta,$$

$$\frac{1}{\omega_\mathrm{c}}\dot{Q}_\mathrm{m} = -Q_\mathrm{m} + Q\ .$$



A Universal & Unified GFM Model

All 3 GFM types can be boiled down to

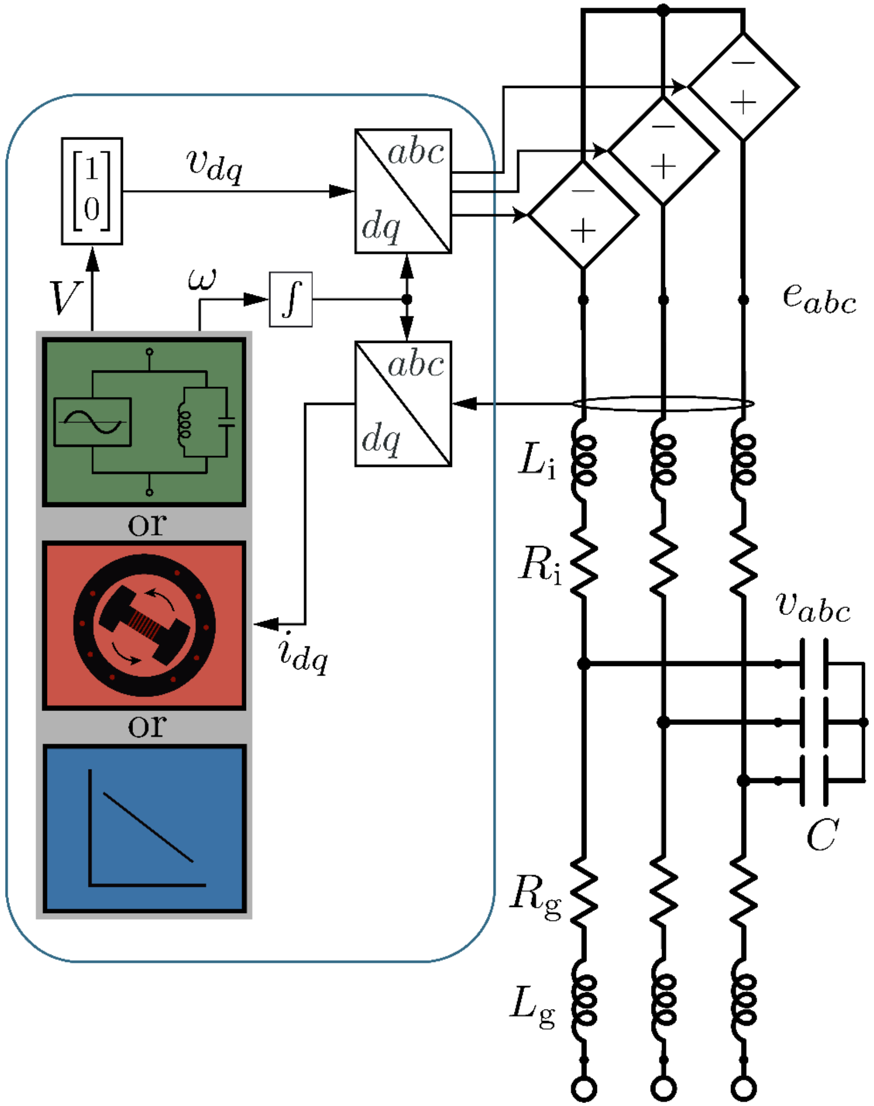
$$\tau_{\mathrm{f}}\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\omega + \omega_0 + \kappa_{\mathrm{d}}\left(\omega_{\mathrm{g}} - \omega\right) + \kappa_{\mathrm{f}}\begin{bmatrix}1 & 0\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}P^\star - P_{\mathrm{m}} \\ Q^\star - Q_{\mathrm{m}}\end{bmatrix},$$

$$\tau_{\mathrm{v}}\frac{\mathrm{d}V}{\mathrm{d}t} = f_{\mathrm{v}}(V) + \kappa_{\mathrm{v}}\begin{bmatrix}0 & 1\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}p^\star - p_{\mathrm{m}} \\ q^\star - q_{\mathrm{m}}\end{bmatrix},$$

$$\frac{1}{\omega_0}\frac{\mathrm{d}\eta}{\mathrm{d}t} = \begin{bmatrix}0 & 1\end{bmatrix}\mathrm{R}(\alpha)\mathrm{R}(\delta)\mathrm{T}(\omega_0 t)v,$$

$$\frac{1}{\omega_0}\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{k_{\mathrm{P}}}{\omega_0}\dot{\eta} + k_{\mathrm{I}}\eta,$$

$$\tau_{\mathrm{p}}\begin{bmatrix}\dot{P}_{\mathrm{m}} \\ \dot{Q}_{\mathrm{m}}\end{bmatrix} = -\begin{bmatrix}P_{\mathrm{m}} \\ Q_{\mathrm{m}}\end{bmatrix} + \begin{bmatrix}P \\ Q\end{bmatrix}.$$



[1] Johnson, et. al, "A generic primary-control model for grid-forming inverters: Towards interoperable operation & control," HICSS, 2021.

[2] Ajala, Baeckeland, Dhople, Dominguez-Garcia, "Uncovering the Kuramoto model from full-order models of grid-forming inverter-based power networks," CDC, 2021.

A Universal & Unified GFM Model

All 3 GFM types can be boiled down to

$$\tau_{\mathrm{f}} \frac{\mathrm{d}\omega}{\mathrm{d}t} = -\omega + \omega_0 + \kappa_{\mathrm{d}}\big(\omega_{\mathrm{g}} - \omega\big) + \kappa_{\mathrm{f}} \begin{bmatrix}1 & 0\end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P_{\mathrm{m}} \\ Q^\star - Q_{\mathrm{m}} \end{bmatrix},$$

$$\tau_{\mathrm{v}} \frac{\mathrm{d}V}{\mathrm{d}t} = f_{\mathrm{v}}(V) + \kappa_{\mathrm{v}} \begin{bmatrix}0 & 1\end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} p^\star - p_{\mathrm{m}} \\ q^\star - q_{\mathrm{m}} \end{bmatrix},$$

$$\frac{1}{\omega_0}\frac{\mathrm{d}\eta}{\mathrm{d}t} = \begin{bmatrix}0 & 1\end{bmatrix} \mathrm{R}(\alpha)\mathrm{R}(\delta)\mathrm{T}(\omega_0 t)v,$$

$$\frac{1}{\omega_0}\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{k_{\mathrm{P}}}{\omega_0}\dot{\eta} + k_{\mathrm{I}}\eta,$$

$$\tau_{\mathrm{p}} \begin{bmatrix} \dot{P}_{\mathrm{m}} \\ \dot{Q}_{\mathrm{m}} \end{bmatrix} = -\begin{bmatrix} P_{\mathrm{m}} \\ Q_{\mathrm{m}} \end{bmatrix} + \begin{bmatrix} P \\ Q \end{bmatrix}.$$

where the parameters are

	τ_{f}	τ_{v}	τ_{p}	κ_{d}	κ_{f}	κ_{v}	$f_{\mathrm{v}}(e^\star)$
droop	0	0	$\frac{1}{\omega_{\mathrm{c}}}$	0	$\frac{1}{d_{\mathrm{f}}}$	$\frac{1}{d_{\mathrm{v}}}$	$-V + V_0$
VSM	J	0	$\frac{1}{\omega_{\mathrm{c}}}$	$\frac{d_{\mathrm{d}}}{d_{\mathrm{f}}}$	$\frac{1}{d_{\mathrm{f}}}$	$\frac{1}{d_{\mathrm{v}}}$	$-V + V_0$
dVOC	0	$\frac{1}{\omega_0}$	0	0	$\frac{\omega_0 \kappa_1}{V^2}$	$\frac{\kappa_1}{V}$	$\kappa_2(-V^2 + V_0^2)V$

[1] Johnson, et. al, "A generic primary-control model for grid-forming inverters: Towards interoperable operation & control," HICSS, 2021.

[2] Ajala, Baeckeland, Dhople, Dominguez-Garcia, "Uncovering the Kuramoto model from full-order models of grid-forming inverter-based power networks," CDC, 2021.

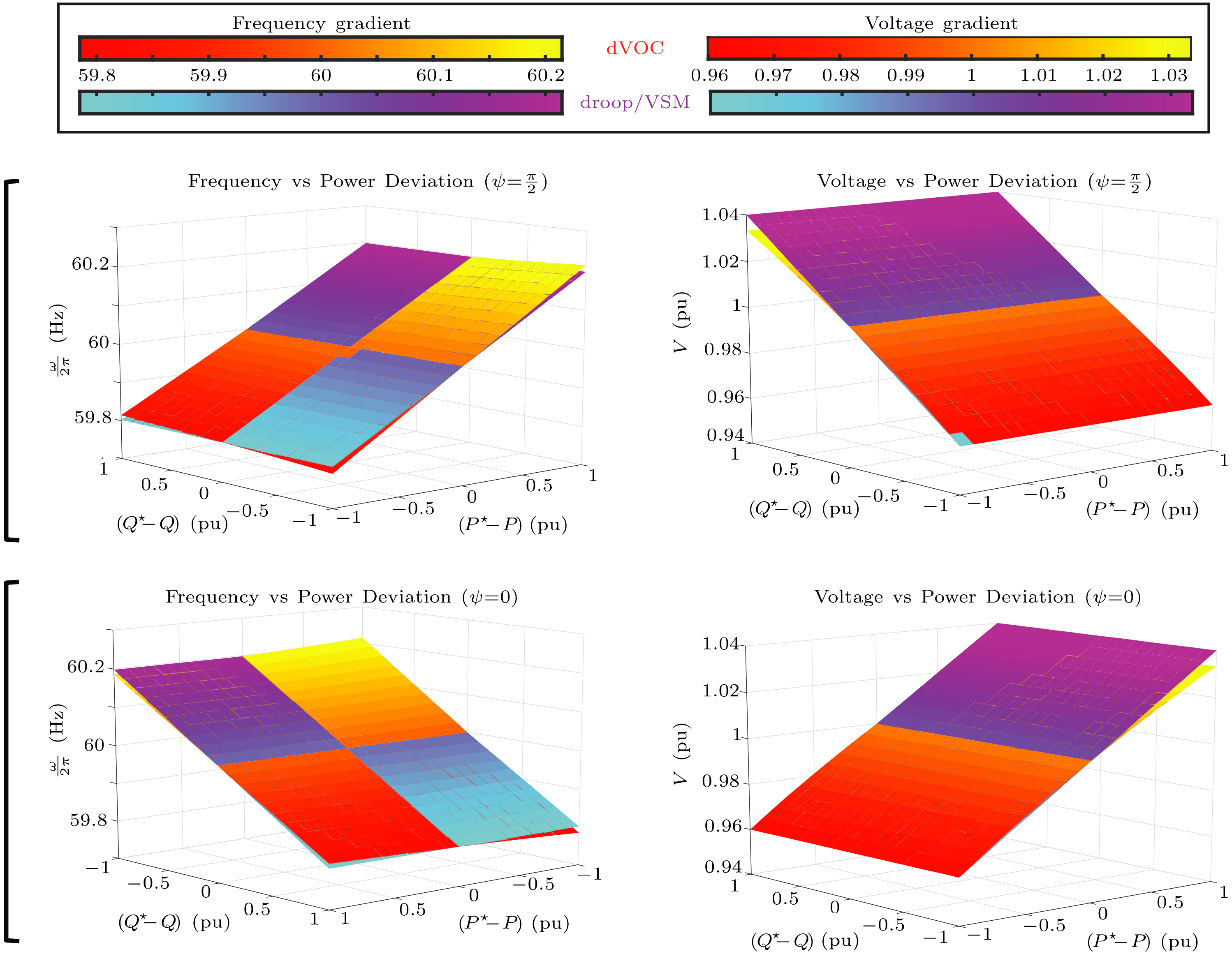
Steady-state Performance of GFM Types

inductive lines

$\psi = \frac{\pi}{2}$

resistive lines

$\psi = 0$



[1] Johnson, et. al, “A generic primary-control model for grid-forming inverters: Towards interoperable operation & control,” HICSS, 2021.

[2] Ajala, Baeckeland, Dhople, Dominguez-Garcia, “Uncovering the Kuramoto model from full-order models of grid-forming inverter-based power networks,” CDC, 2021.

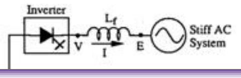
Unifying 25+ Years of GFM Technologies

1993

Control of Parallel Connected Inverters in Standalone ac Supply Systems

Mukul C. Chandorkar, *Student Member, IEEE*, Deepakraj M. Divan, *Member, IEEE*, and Rambabu Adapa, *Senior Member, IEEE*

***Abstract*—A scheme for controlling parallel-connected inverters in a standalone ac supply system is presented in this paper. This scheme is suitable for control of inverters in distributed source environments such as in isolated ac systems, large and**



2014

Synchronization of Parallel Single-Phase Inverters With Virtual Oscillator Control

Brian B. Johnson, *Member, IEEE*, Sairaj V. Dhople, *Member, IEEE*, Abdullah O. Hamadeh, and Philip T. Krein, *Fellow, IEEE*

***Abstract*—A method to synchronize and control a system of parallel single-phase inverters without communication is presented. Inspired by the phenomenon of synchronization in networks of coupled oscillators, we propose that each inverter be controlled** proportion to their ratings. Focused on these challenges, this paper presents a method to synchronize and control a system of parallel single-phase inverters without communication. It is

2007

Virtual Synchronous Machine

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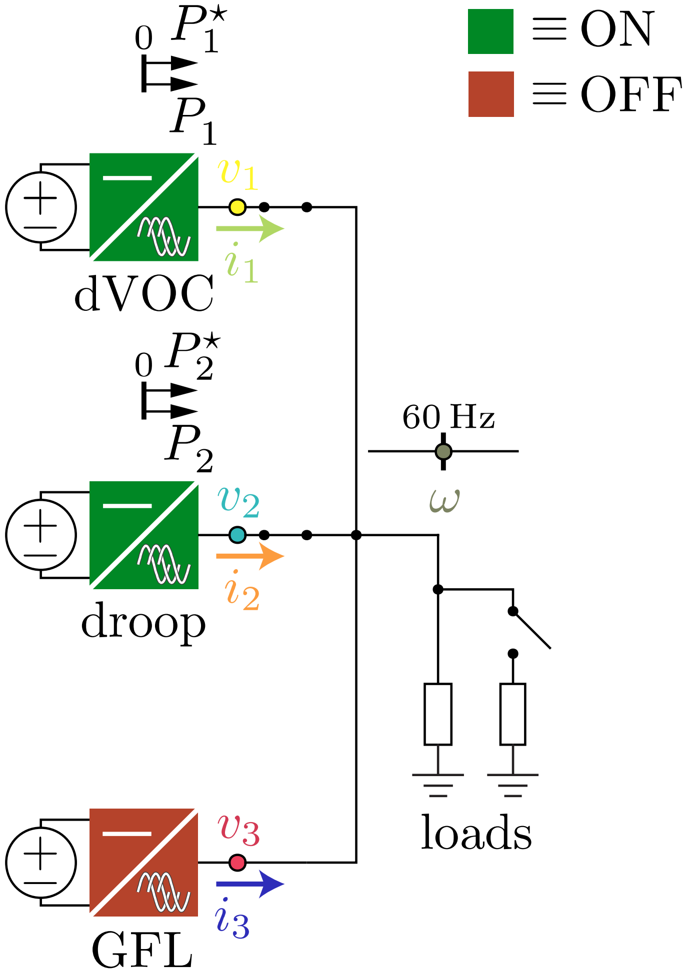
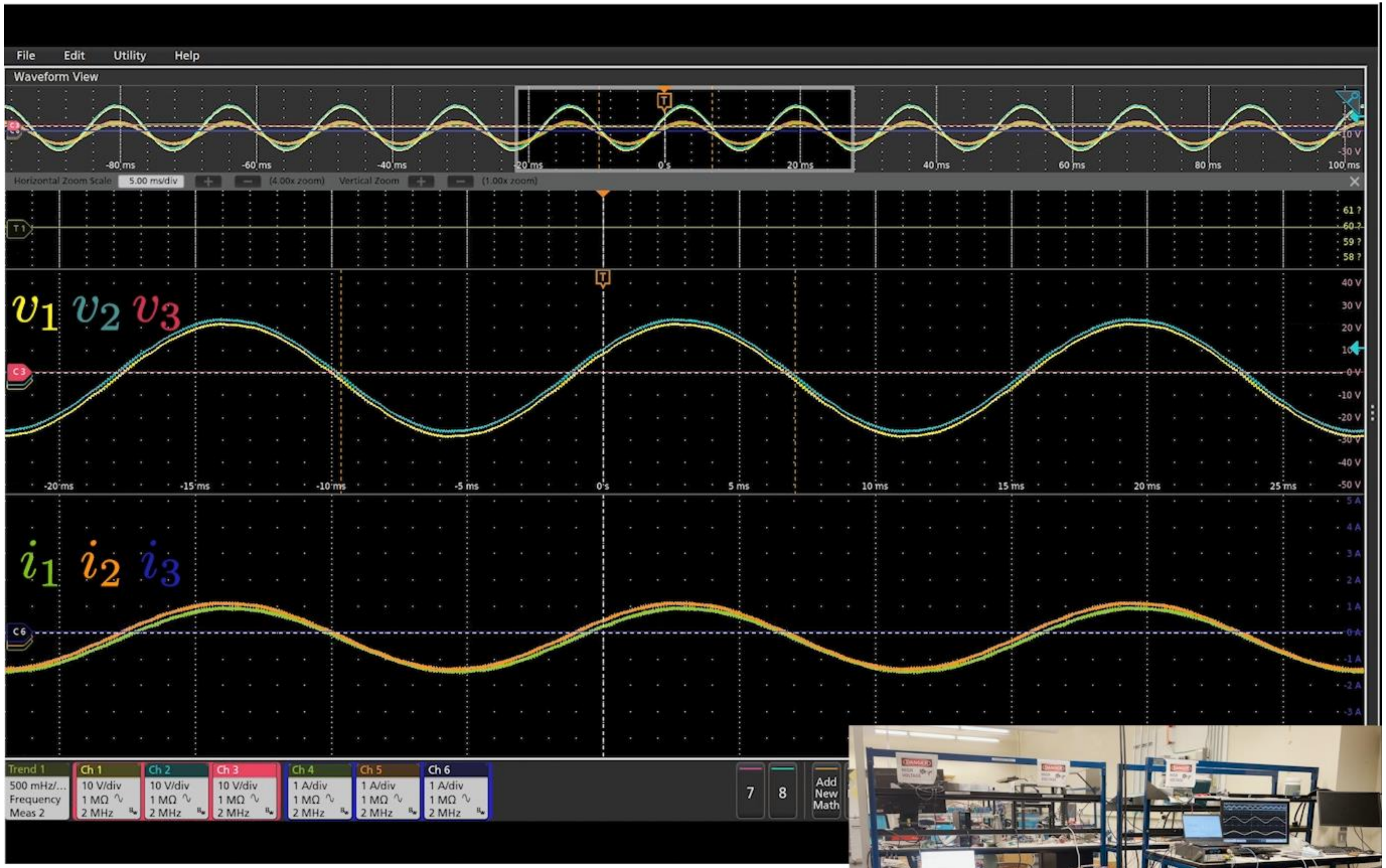
Demands in the area of electrical energy generation and distribution, as a result of energy policies, are leading to far reaching changes in the structure of the energy supply, which is cause or promote the appearance and mitigation of various disturbances.

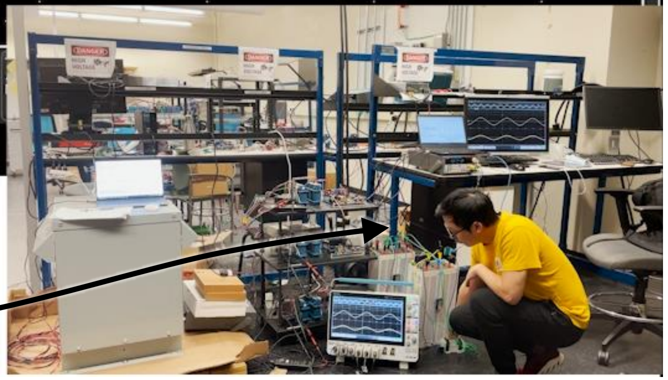
$$
\begin{aligned}
\tau_{\mathrm{f}} \frac{\mathrm{d}\omega}{\mathrm{d}t} &= -\omega + \omega_0 + \kappa_{\mathrm{d}}(\omega_{\mathrm{g}} - \omega) \\
&\quad + \kappa_{\mathrm{f}} \begin{bmatrix} 1 & 0 \end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P_{\mathrm{m}} \\ Q^\star - Q_{\mathrm{m}} \end{bmatrix}, \\
\tau_{\mathrm{v}} \frac{\mathrm{d}V}{\mathrm{d}t} &= f_{\mathrm{v}}(V) + \kappa_{\mathrm{v}} \begin{bmatrix} 0 & 1 \end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} p^\star - p_{\mathrm{m}} \\ q^\star - q_{\mathrm{m}} \end{bmatrix}, \\
\frac{1}{\omega_0} \frac{\mathrm{d}\eta}{\mathrm{d}t} &= \begin{bmatrix} 0 & 1 \end{bmatrix} \mathrm{R}(\alpha)\mathrm{R}(\delta)\mathrm{T}(\omega_0 t) v, \\
\frac{1}{\omega_0} \frac{\mathrm{d}\alpha}{\mathrm{d}t} &= \frac{k_{\mathrm{P}}}{\omega_0} \dot{\eta} + k_{\mathrm{I}} \eta, \\
\tau_{\mathrm{p}} \begin{bmatrix} \dot{P}_{\mathrm{m}} \\ \dot{Q}_{\mathrm{m}} \end{bmatrix} &= -\begin{bmatrix} P_{\mathrm{m}} \\ Q_{\mathrm{m}} \end{bmatrix} + \begin{bmatrix} P \\ Q \end{bmatrix},
\end{aligned}
$$

A unified model

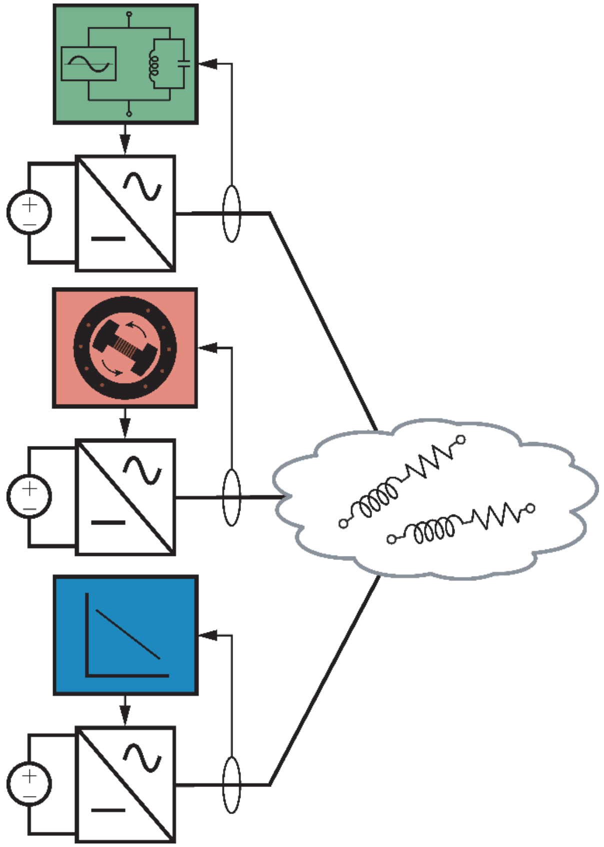
Showing Interoperability with a Mix of Control Types



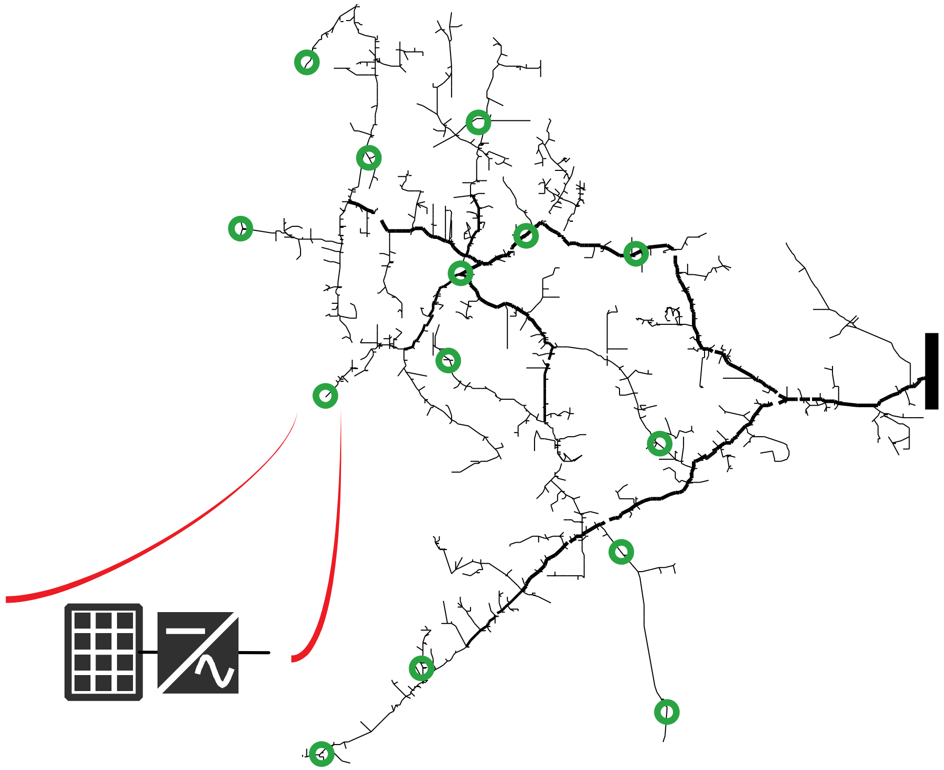




Weiqian Cai



Interoperability



Self-organizing GFM Networks

A Single-phase GFM Commercial Product

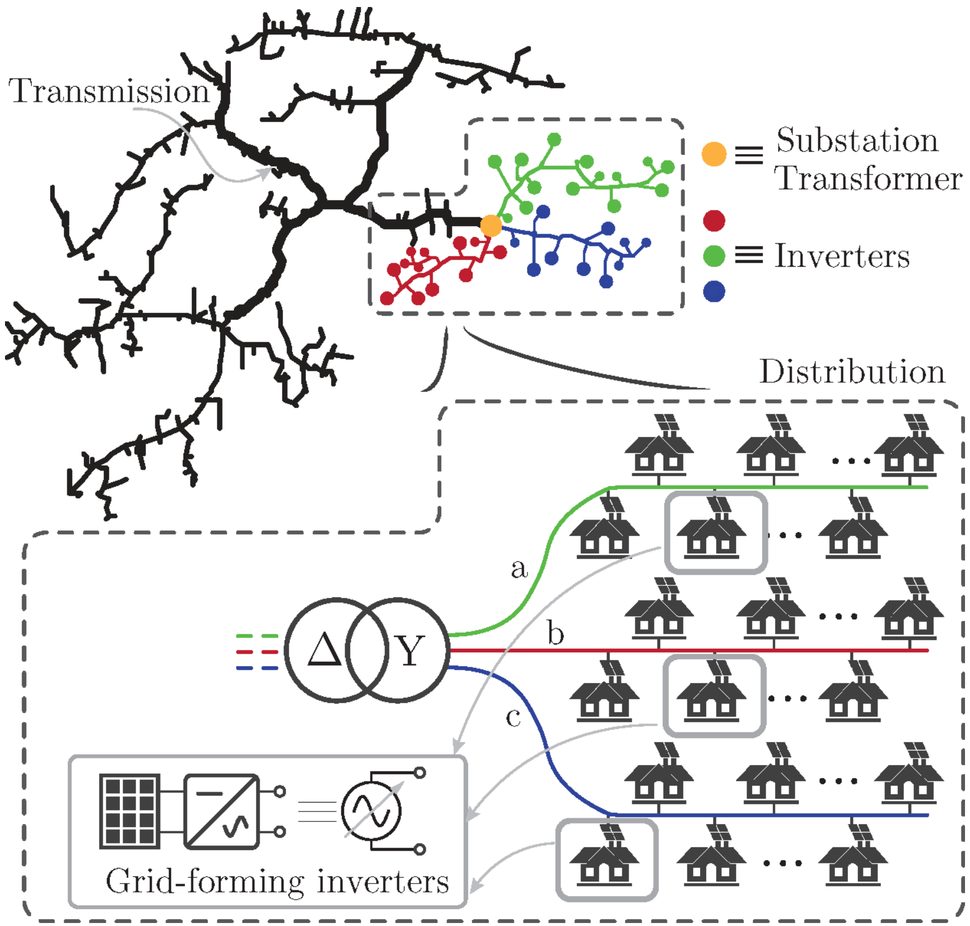
Features of the inverter building block

- 300 VA single-phase with droop-based GFM controls
- Bidirectional converter can interface PV or batteries

Over 39M+ Enphase inverters shipped for 12 GW of capacity as of September 2021

Building Three-phase Systems with Single-phase GFM Units

Can swarms of decentralized single-phase GFMs self-organize into a three-phase system?



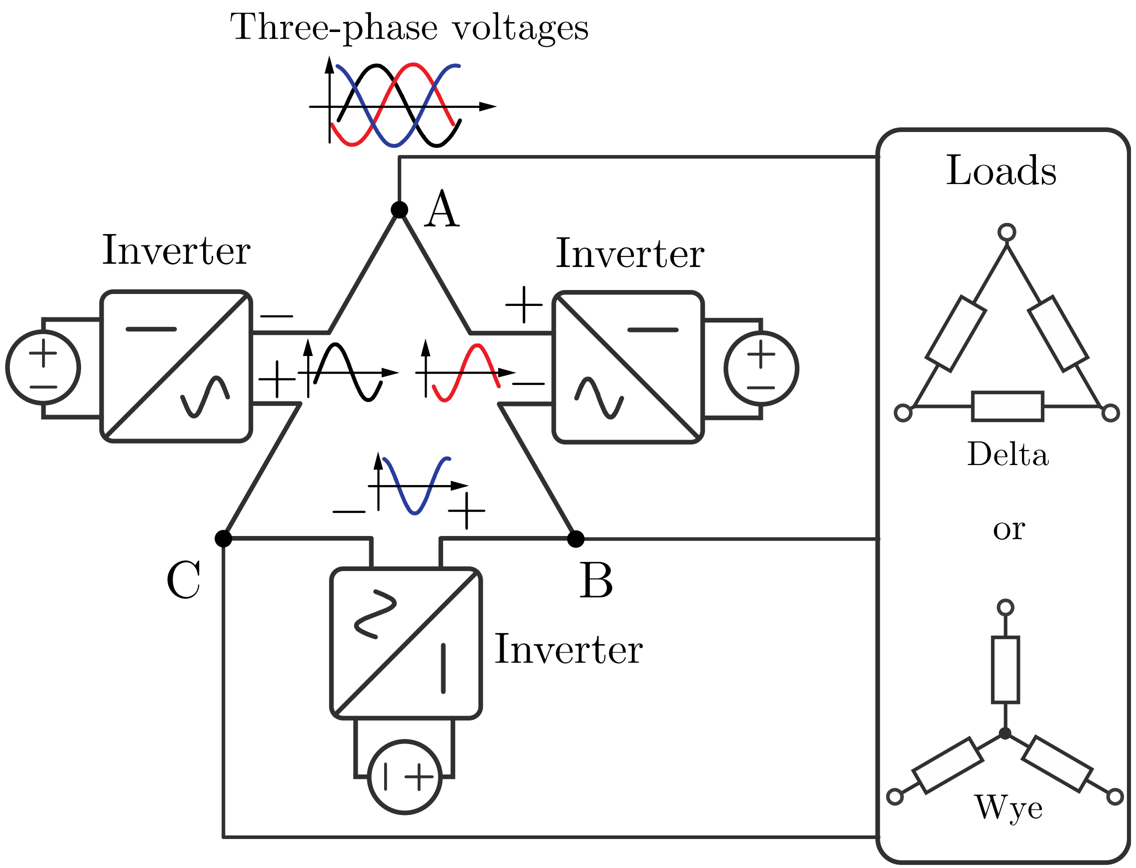
Could this system maintain phase balancing when islanded?

From Industry Challenges to Foundational Research

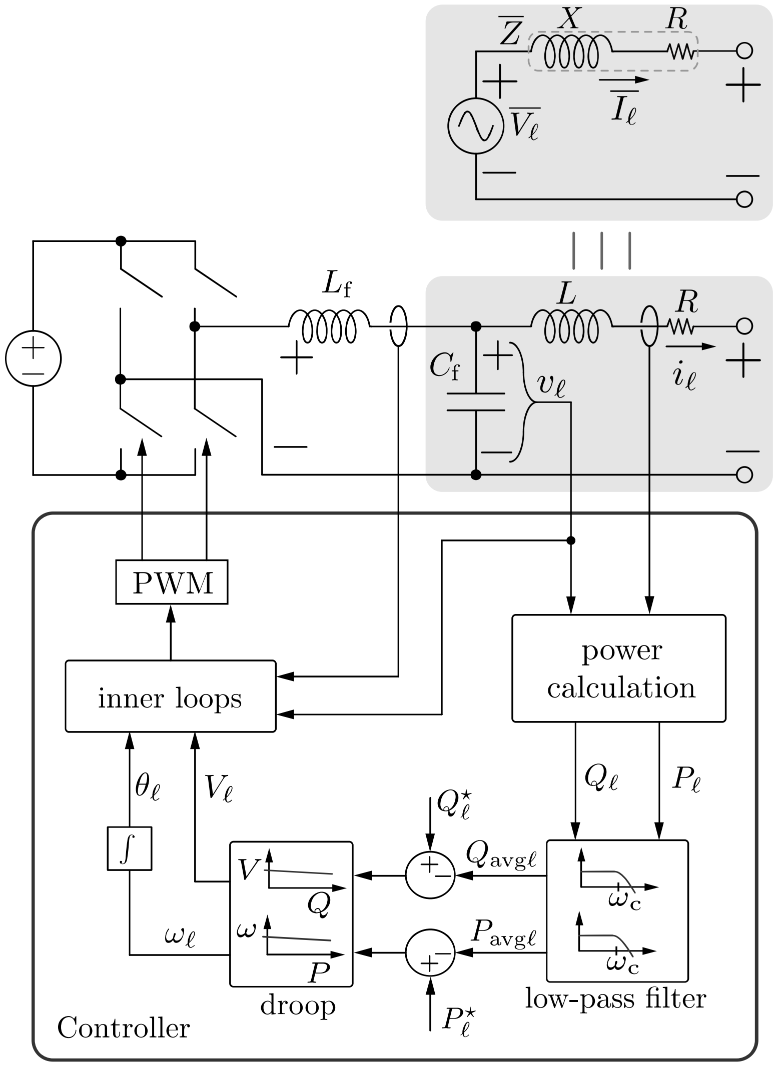
Industry partners have already prioritized three-phase system operation with single-phase units

ENPHASE.

Donny Zimmanck



Single-phase Inverter Model



A single-phase droop-controlled inverter

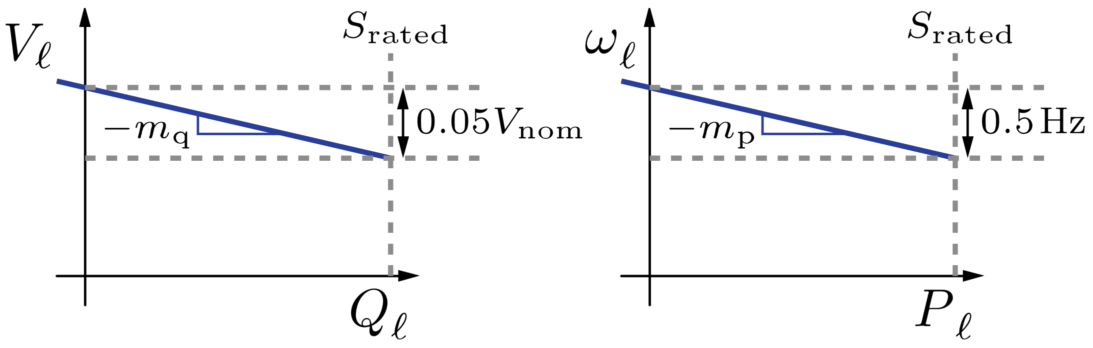
The $\ell \in 1, 2, 3$ inverter has the droop laws

$$V_\ell = V_{\mathrm{nom}} - m_{\mathrm{q}} \left(Q_{\mathrm{avg},\ell} - Q_\ell^\star \right),$$
$$\omega_\ell = \omega_{\mathrm{nom}} - m_{\mathrm{p}} \left(P_{\mathrm{avg},\ell} - P_\ell^\star \right),$$

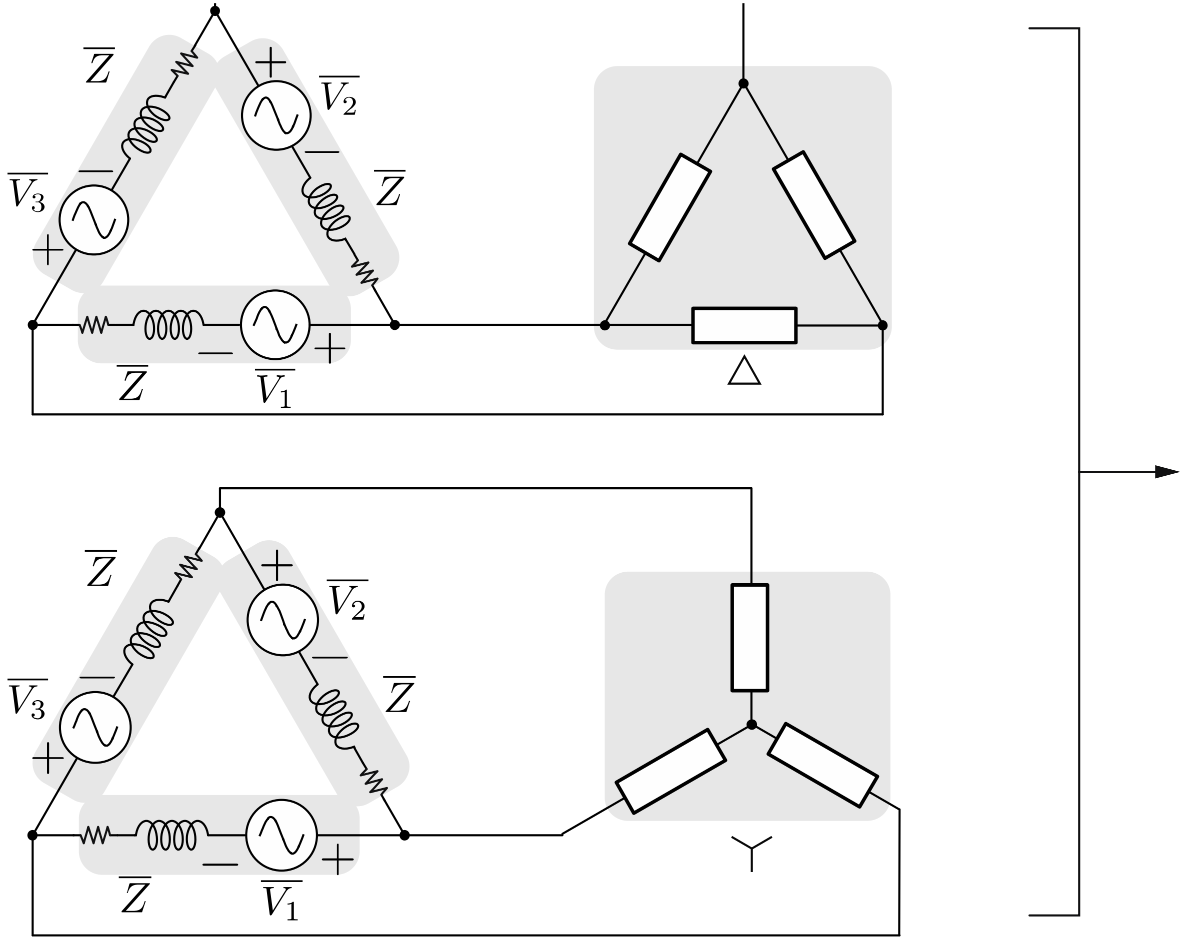
where

$$\dot{P}_{\mathrm{avg},\ell} = \omega_{\mathrm{c}} \left(P_\ell - P_{\mathrm{avg},\ell} \right),$$
$$\dot{Q}_{\mathrm{avg},\ell} = \omega_{\mathrm{c}} \left(Q_\ell - Q_{\mathrm{avg},\ell} \right),$$

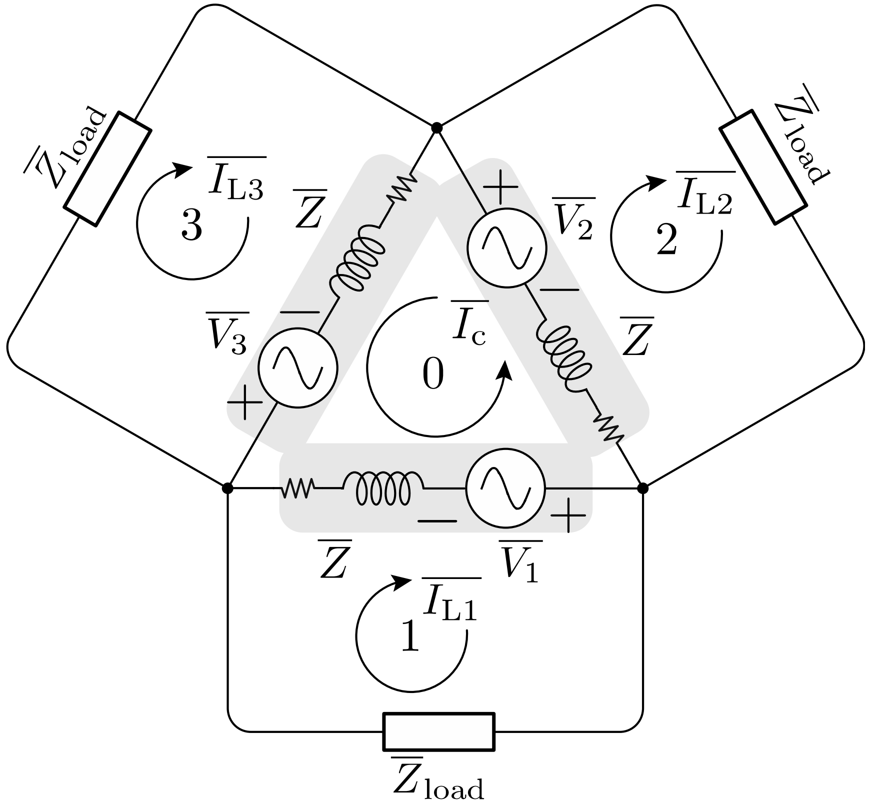
and slopes $m_{\mathrm{q}} = 0.05 \frac{V_{\mathrm{nom}}}{S_{\mathrm{rated}}}$, $m_{\mathrm{p}} = \frac{2\pi \times 0.5}{S_{\mathrm{rated}}}$. This gives the following in steady-state:



A Common Framework For Various Load Configurations



inverters with $\triangle$ & $\curlyvee$ loads



circuit model with mesh loops

A System Circuit Model

Circuit laws + control dynamics give

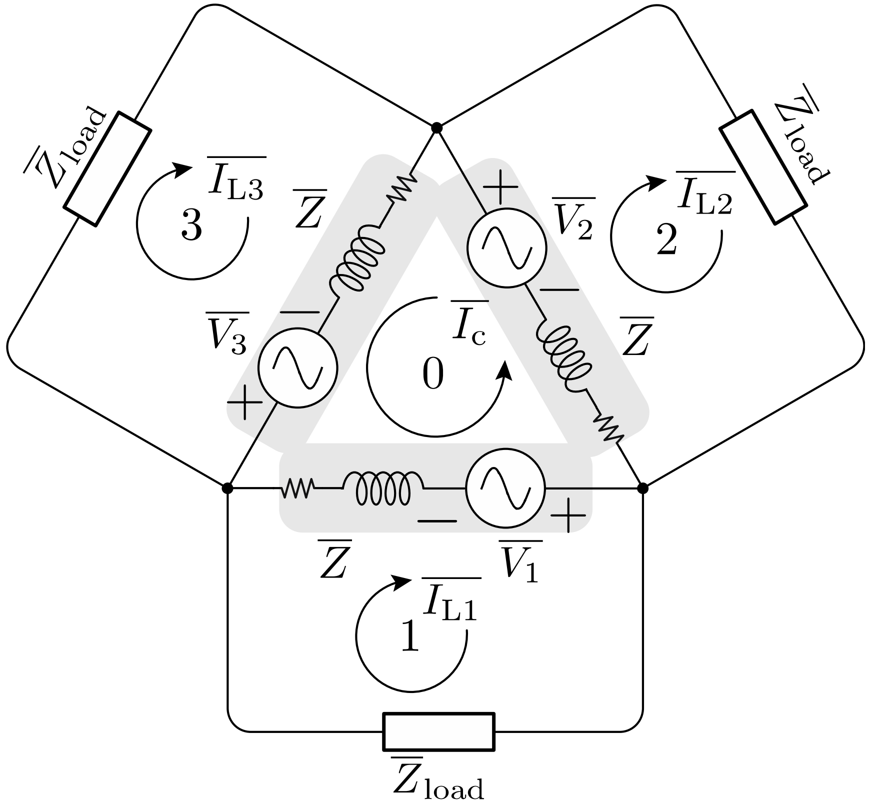
$$V_\ell = V_{\mathrm{nom}} - m_{\mathrm{q}} \sum_{k=1}^{3} \frac{V_k V_\ell}{|\overline{Z}_{\mathrm{loop}}|} \sin(\theta_{k\ell} - \phi),$$

$$\dot{\theta}_\ell = \omega_{\mathrm{nom}} - m_{\mathrm{p}} \sum_{k=1}^{3} \frac{V_k V_\ell}{|\overline{Z}_{\mathrm{loop}}|} \cos(\theta_{k\ell} - \phi).$$

where $\overline{Z}_{\mathrm{loop}} = 3\overline{Z}$ and $\phi = \tan^{-1}(X/R)$. Next, focus on angle difference dynamics

$$\dot{\theta}_{21} = \dot{\theta}_2 - \dot{\theta}_1,$$

$$\dot{\theta}_{31} = \dot{\theta}_3 - \dot{\theta}_1.$$



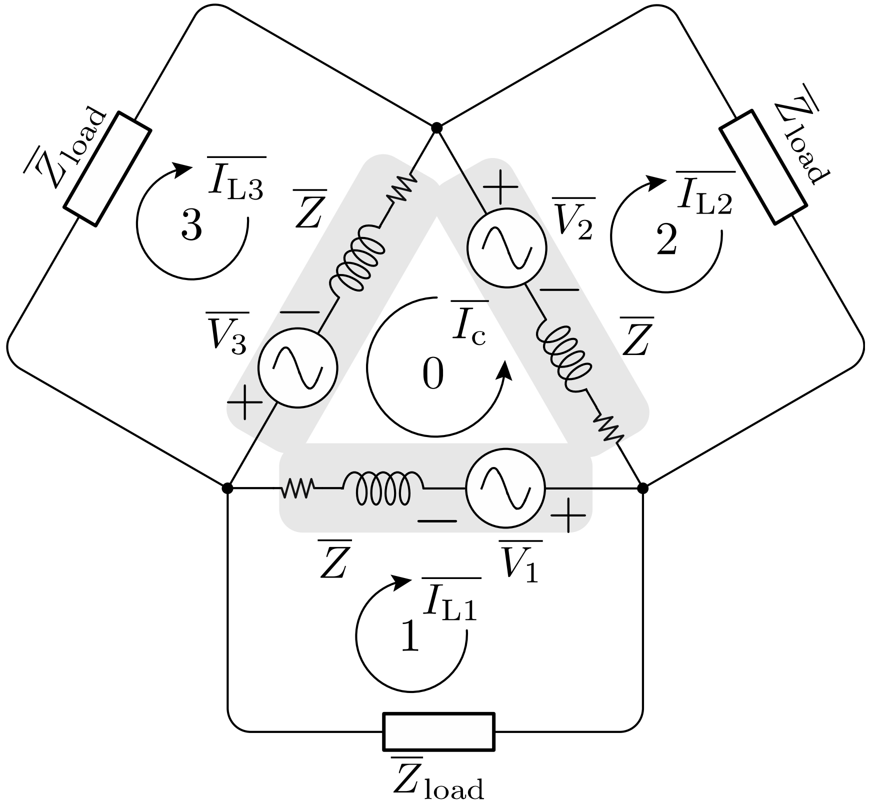
circuit model with mesh loops

Nonlinear Angular Dynamics

Algebra gives us

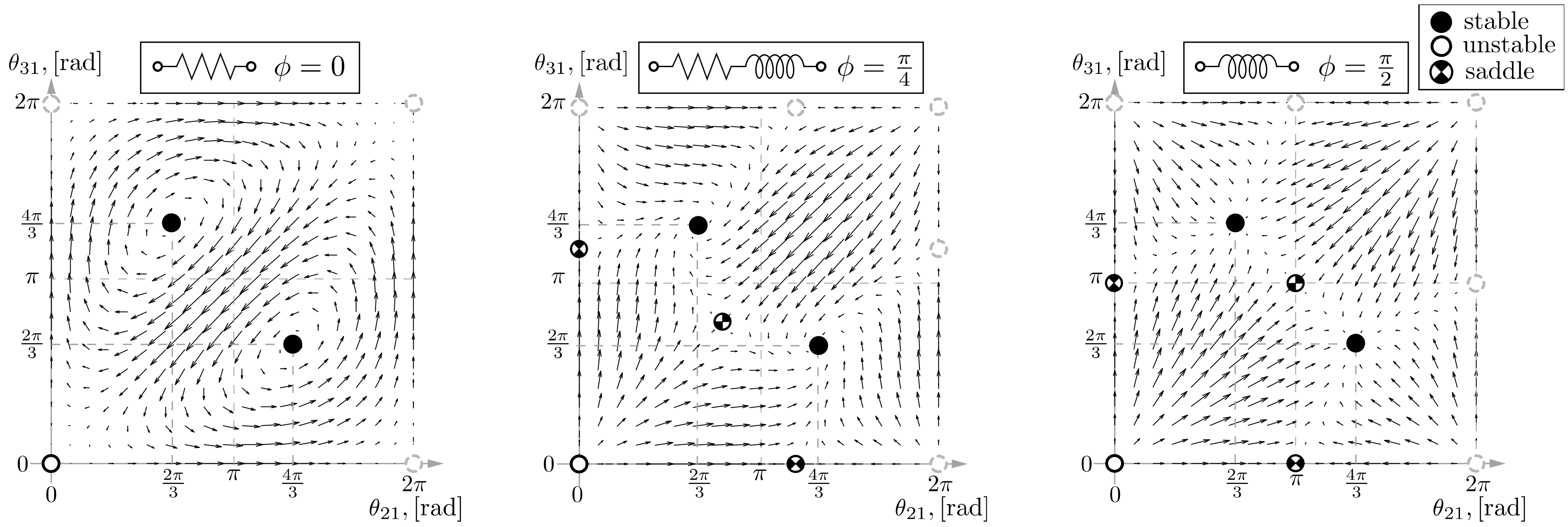
$$\dot{\theta}_{21} = \frac{m_{\mathrm{p}} V_2 V_1}{|\overline{Z}_{\mathrm{loop}}|} \cos(\theta_{21} - \phi) + \frac{m_{\mathrm{p}} V_3 V_1}{|\overline{Z}_{\mathrm{loop}}|} \cos(\theta_{31} - \phi) - \frac{m_{\mathrm{p}} V_2 V_3}{|\overline{Z}_{\mathrm{loop}}|} \cos(\theta_{32} - \phi) - \frac{m_{\mathrm{p}} V_2 V_1}{|\overline{Z}_{\mathrm{loop}}|} \cos(\theta_{12} - \phi),$$

$$\dot{\theta}_{31} = \frac{m_{\mathrm{p}} V_3 V_1}{|\overline{Z}_{\mathrm{loop}}|} \cos(\theta_{31} - \phi) + \frac{m_{\mathrm{p}} V_2 V_1}{|\overline{Z}_{\mathrm{loop}}|} \cos(\theta_{21} - \phi) + \frac{m_{\mathrm{p}} V_2 V_3}{|\overline{Z}_{\mathrm{loop}}|} \cos(\theta_{23} - \phi) + \frac{m_{\mathrm{p}} V_1 V_3}{|\overline{Z}_{\mathrm{loop}}|} \cos(\theta_{13} - \phi).$$



circuit model with mesh loops

Angle Difference Dynamics



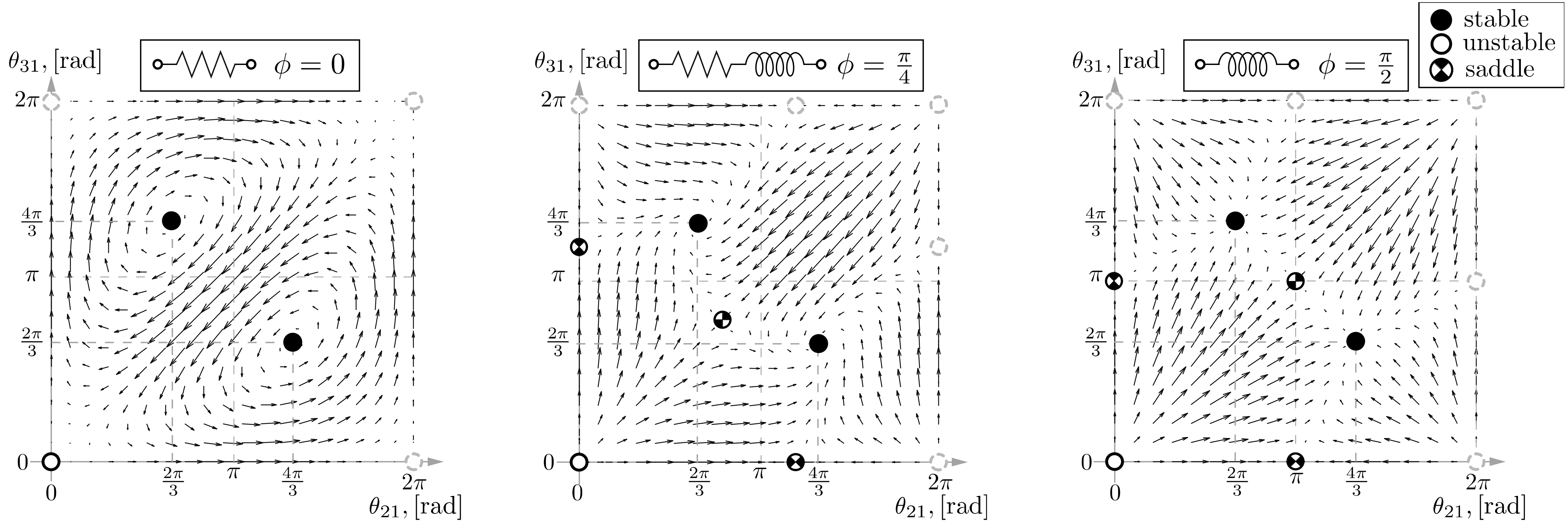
Substitute voltage droop laws, ignore $m_{\mathrm{p}}^2, m_{\mathrm{q}}^2, m_{\mathrm{p}} m_{\mathrm{q}}$ terms, and let $K = m_{\mathrm{p}} V_{\mathrm{nom}}^2 / |\overline{Z}_{\mathrm{loop}}|$ to get:

$$\dot{\theta}_{21} \approx K(2\sin\theta_{21} + \sin\theta_{31} + \sin(\theta_{21} - \theta_{31}))\sin\phi + K(\cos\theta_{31} - \cos(\theta_{21} - \theta_{31}))\cos\phi,$$

$$\dot{\theta}_{31} \approx K(2\sin\theta_{31} + \sin\theta_{21} + \sin(\theta_{31} - \theta_{21}))\sin\phi + K(\cos\theta_{21} - \cos(\theta_{31} - \theta_{21}))\cos\phi.$$

[1] Lu, Dhople, Zimmanck, Johnson, "Spontaneous phase balancing in delta-connected single-phase grid-forming inverters," TPEL, 2021.

Angle Difference Dynamics



The equilibria are below, where $\sigma_1 = 2\pi + 2\tan^{-1}(-3\tan\phi)$, $\sigma_2 = 2\tan^{-1}(3\tan\phi)$.

$$(\theta_{21,\mathrm{eq}},\ \theta_{31,\mathrm{eq}})^{\bullet} = \left\{\left(\tfrac{2\pi}{3},\ \tfrac{4\pi}{3}\right),\left(\tfrac{4\pi}{3},\ \tfrac{2\pi}{3}\right)\right\},$$

$$(\theta_{21,\mathrm{eq}},\ \theta_{31,\mathrm{eq}})^{\circ} = \{(0,0)\},$$

$$(\theta_{21,\mathrm{eq}},\ \theta_{31,\mathrm{eq}})^{\otimes} = \{(0,\sigma_1),(\sigma_1,0),\ (\sigma_2,\sigma_2)\},$$

[1] Lu, Dhople, Zimmanck, Johnson, “Spontaneous phase balancing in delta-connected single-phase grid-forming inverters,” TPEL, 2021.

Small-signal Modeling

The linearized system is

$$\begin{bmatrix}\Delta\dot{\theta}_{21}\\ \Delta\dot{\theta}_{31}\end{bmatrix} = \mathcal{J}(\theta_{21,\mathrm{eq}},\theta_{31,\mathrm{eq}})\begin{bmatrix}\Delta\theta_{21}\\ \Delta\theta_{31}\end{bmatrix},$$

where

$$\mathcal{J}(\theta_{21,\mathrm{eq}},\theta_{31,\mathrm{eq}}) = K\left[\begin{array}{c|c} (2\cos\theta_{21,\mathrm{eq}}+\cos(\theta_{21,\mathrm{eq}}-\theta_{31,\mathrm{eq}}))\sin\phi & (\cos\theta_{31,\mathrm{eq}}-\cos(\theta_{21,\mathrm{eq}}-\theta_{31,\mathrm{eq}}))\sin\phi \\ +\sin(\theta_{21,\mathrm{eq}}-\theta_{31,\mathrm{eq}})\cos\phi & +(\sin(\theta_{31,\mathrm{eq}}-\theta_{21,\mathrm{eq}})-\sin\theta_{31,\mathrm{eq}})\cos\phi \\ \hline (\cos\theta_{21,\mathrm{eq}}-\cos(\theta_{31,\mathrm{eq}}-\theta_{21,\mathrm{eq}}))\sin\phi & (2\cos\theta_{31,\mathrm{eq}}+\cos(\theta_{31,\mathrm{eq}}-\theta_{21,\mathrm{eq}}))\sin\phi \\ +(\sin(\theta_{21,\mathrm{eq}}-\theta_{31,\mathrm{eq}})-\sin\theta_{21,\mathrm{eq}})\cos\phi & +\sin(\theta_{31,\mathrm{eq}}-\theta_{21,\mathrm{eq}})\cos\phi \end{array}\right],$$

and the eigenvalues for all equilibria within $\phi \in (0, \pi/2]$ are

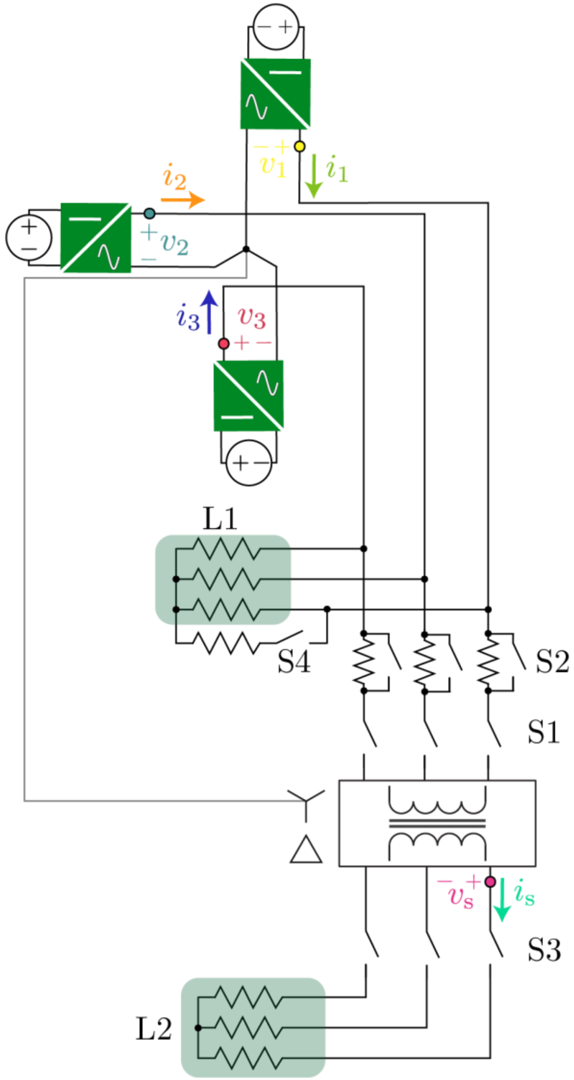
$$\lambda_{1,2}^{\bullet} = -\frac{3}{2}K(\sin\phi \pm \jmath\cos\phi), \longrightarrow \Re(\lambda_{1,2}^{\bullet}) < 0 \longrightarrow \text{stable,}$$

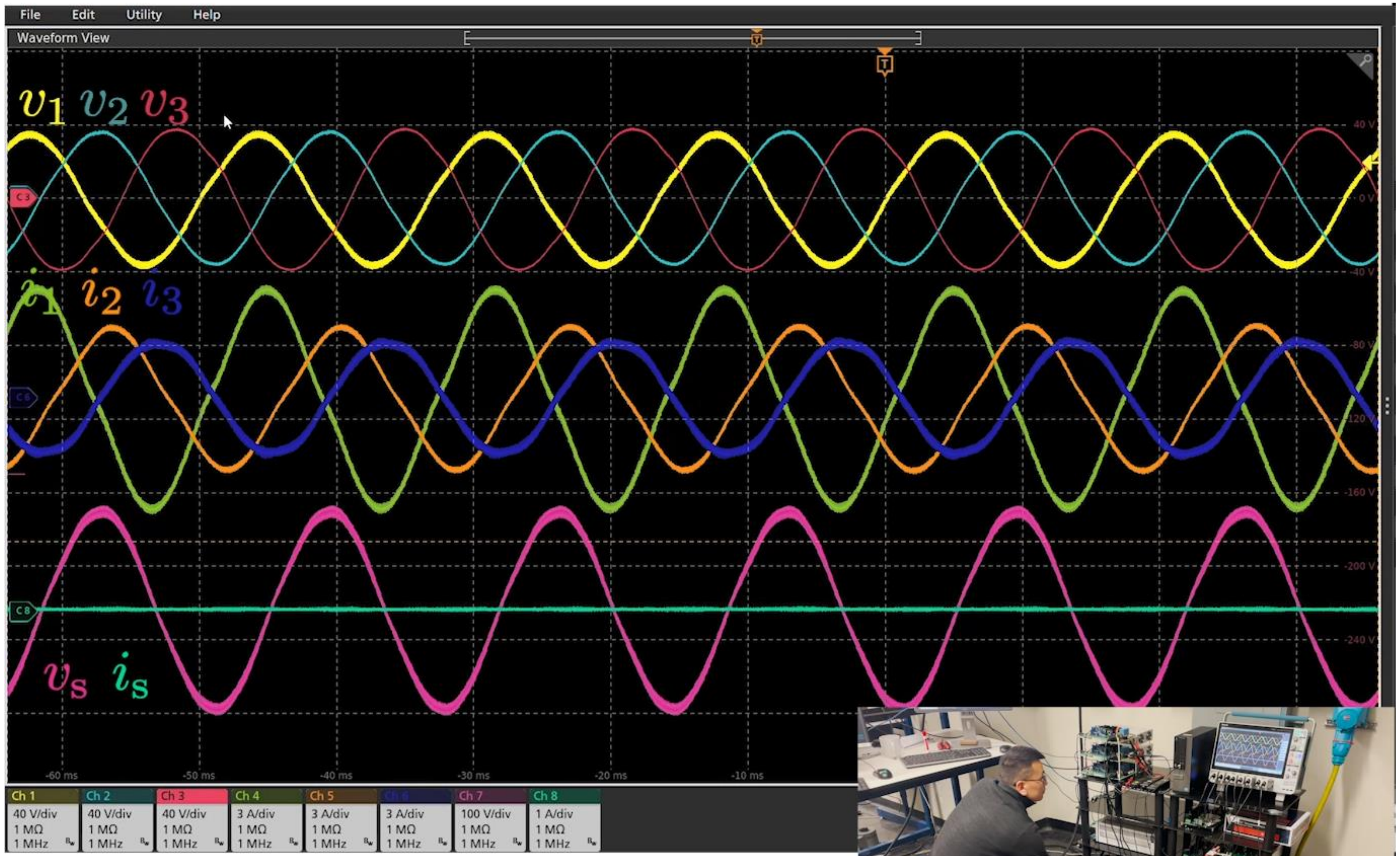
$$\lambda_{1,2}^{\circ} = 3K\sin\phi, \longrightarrow \Re(\lambda_{1,2}^{\circ}) > 0 \longrightarrow \text{unstable,}$$

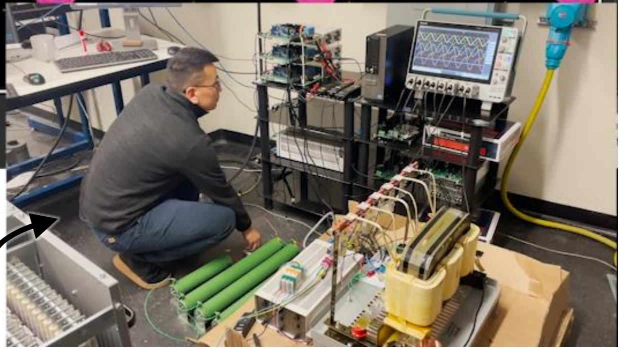
$$\lambda_{1}^{\oplus} = -3K\sin\phi, \quad \lambda_{2}^{\oplus} = 9K\frac{1+\tan^2\phi}{1+9\tan^2\phi}\sin\phi, \longrightarrow \Re(\lambda_{1}^{\oplus}) < 0 \text{ and } \Re(\lambda_{2}^{\oplus}) > 0 \longrightarrow \text{saddle.}$$

[1] Lu, Dhople, Zimmanck, Johnson, “Spontaneous phase balancing in delta-connected single-phase grid-forming inverters,” TPEL, 2021.

Self-balancing Single-phase GFM Hardware Results



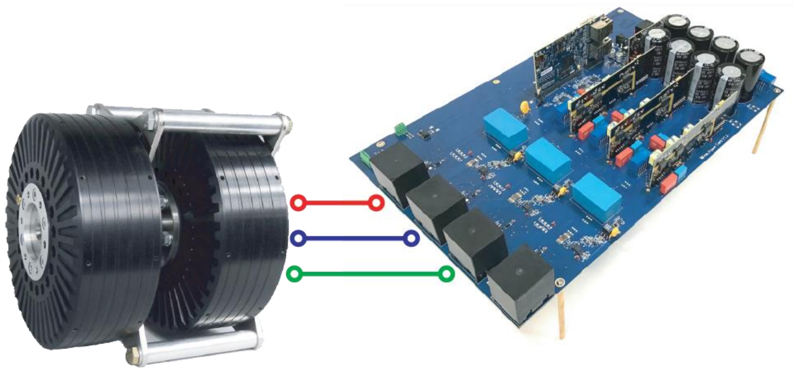




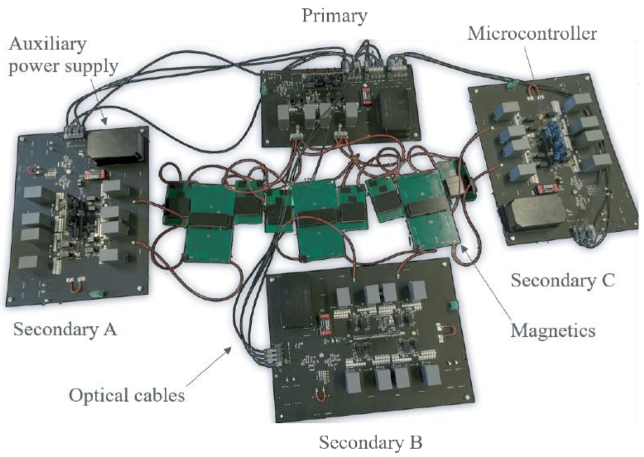
Dr. Minghui Lu

Thanks for Your Attention!

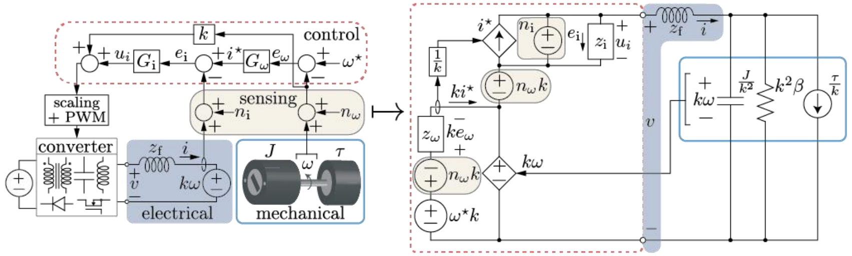
My other work building & analyzing future energy systems at all scales



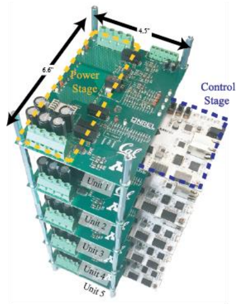
electromechanics



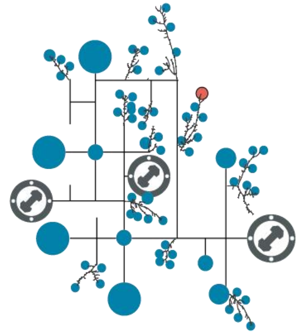
medium-voltage electronics



multiphysics modeling



dc-dc converters



system dynamics